Forecasting Dutch GDP using large scale models

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Outline of the presentation

- Introduction
  - Motivation
  - philosophy
- The model
  - Dynamic vs. Static specification
- The data
  - Data underlying structural model + leading indicator variables
- Results
  - Both model and data selection matters

Motivation

- Forecasting models:
  - Structural macro models (MORKMON)
  - dynamic stochastic general equilibrium models
  - indicator models: typically univariate or low-order VAR

- Policy makers and applied forecasters are generally keen to extract information from many more sources and data describing economic activity at a more disaggregated level.

- Some asset prices have been useful predictors of inflation and/or output growth in some countries in some time periods. However, exploitation requires *a priori* knowledge what asset price works when in which country. Stock & Watson (2003)

Factor models: philosophy

- Essential characteristics of macroeconomic motions are captured by a few driving aggregate forces and that the information contained in all potentially available economic key variables at an aggregate level are individually less informative about macroeconomic behaviour.

- Large cross-sections
  - For forecasting multivariate information helps
  - multi-region and/or sectoral analysis: propagation of shocks
  - Insurance against model uncertainty and variable selection

- Factor models exploit in a parsimonious way the information contained in large cross-sections, without imposing too many restrictions

Factor models approaches

- Few sources of common dynamics
  - Sargent & Sims (1977) and Geweke (1977)
  - Stock & Watson (1989) formalization of a single factor driving the four coincident variabels used by the Conference Board
- Approaches in the recent literature
  - Stock & Watson (2002, 2002) use principal component based estimator of the factors
  - Forni *et al*. (2000) use dynamic principal components and apply frequency domain techniques
- This paper compares both methods

The model:

- Denote by $\mathbf{x}_{nt} = (x_{1t} \dots x_{nt})'$ an *nxT* rectangular array of *T* observations from *n* demeaned stationary processes

- every variable split into common and idiosyncratic unobservable components: $x_{it} = \chi_{it} + \xi_{it}$

- The common component is driven by a *q*-dimensional set of orthonormal white noise processes (*f_{1t}*, …, *f_{qt}*):

$$\chi_{it} = b_{i1}(L) f_{1t} + \dots + b_{iq}(L) f_{qt} = \sum_{l=0}^{s} \sum_{j=1}^{q} b_{ij}(l) f_{j(t-l)}$$

- Defining: $\boldsymbol{\xi}_{nt} = (\xi_{1t}, \dots, \xi_{nt})'$ and $\mathbf{B}_n(L) = (b_{i1}(L), \dots, b_{iq}(L))'$

- The model becomes $\mathbf{x}_{nt} = \boldsymbol{\Lambda}_n \mathbf{F}_t + \boldsymbol{\xi}_{nt} = \boldsymbol{\chi}_{nt} + \boldsymbol{\xi}_{nt} = \mathbf{B}_n(L)\mathbf{f}_t + \boldsymbol{\xi}_{nt}$

- With $F_t = (\mathbf{f}_t' \ \mathbf{f}_{t-1}' \dots \mathbf{f}_{t-s}')'$ and $\boldsymbol{\Lambda}_n = (\mathbf{B}_0^n \ \mathbf{B}_1^n \ \dots \ \mathbf{B}_s^n)$

- Involving *r=q(s+1)* static factors

Model - properties

- For every factor model, the common and idiosyncratic components are orthogonal: $\forall (t, \tau) \ E\left(\mathrm{f}_t \xi'_{n\tau}\right) = 0.$

Factors can be identified only up to an orthogonal transformation:
$\mathbf{\Lambda F} = \mathbf{\Lambda Q Q}^{-1}\mathbf{F} = \mathbf{\Lambda}^* \mathbf{F}^*.$ for a non-singular matrix Q

- $Var(F_t) = I$ and therefore $Var(X) = \Sigma = \Lambda\Lambda' + D$
  - If the data approves this property, then principal components estimator can be used to estimate the unobservable factors

- Classical static factor model assumes mutual orthogonality of idiosyncratic components:

$$\forall (i, j), \forall (t, \tau) \ E\left(\boldsymbol{\xi}_{it} \boldsymbol{\xi}'_{j\tau}\right) = \begin{cases} 0 \text{ if } t \neq \tau \text{ or } i \neq j \\ diag\left(d_1, ..., d_n\right) \text{ otherwise} \end{cases}$$

Model – properties (2)

- Limited cross-sectional and auto-correlation allowed for by approximate factor model (Arbitrage Pricing Theory)

    - The double orthogonality assumption is replaced by respectively divergence and boundedness of the eigenvalues μ: $\mu_{n,r}^{\chi} \to \infty \text{ as } n \to \infty \qquad \mu_{n,k}^{\xi} \le M \text{ , } \forall \text{n}$

- note that static factor matrix Ft possesses special structure:
    - Rank(F_t)=r>q, which is the rank of the spectral density matrix of F_t

Dynamic versus Static Factor model

- A little model to understand the difference. Assume:

$$\chi_{1t} = f_t, \chi_{2t} = f_{t-1}, \mathrm{var}(\xi_{1t}) = \mathrm{var}(\xi_{2t}) = 1, \mathrm{cov}(\xi_{1t}, \xi_{2t}) = 0$$

  - so idiosyncratic-to-common variance ratio = 1 for both variables

- Now the contemporaneous average is:

$$\left(f_t + f_{t-1} + \xi_{1t} + \xi_{2t}\right)/\,2$$

  - so the idiosyncratic-to-common variance ratio is still 1

- If we can manage to shift the x's over time before averaging:

$$\left(x_{1,t-1} + x_{2t}\right)/\,2 = f_{t-1} + \left(\xi_{1t-1} + \xi_{2t}\right)/\,2$$

  - So the idiosyncratic-to-common variance ratio is only 0.5

Dutch dataset

- Morkmon-dataset (five sectors of the economy)
- Supplemented
  - to broaden the dataset: industrial production, external developments
  - with variables of forward looking nature: surveys, order positions, etc.
  - with disaggregated financial/monetary-variables: interest rates, commodity prices, equity prices, exchange rates
- Balanced representation of Dutch macro-economy
  - Common shocks hit all variables
  - Idiosyncratic shocks only hit subgroups (like sectors) and die out in the aggregate
- Data treatment: TRAMO and stationarity inducing transformations (mostly first differences of logs)
- Final dataset, N=370, T=1980Q1-2002Q4
- Note: 2004q2 snapshot of the data, ignoring data revisions

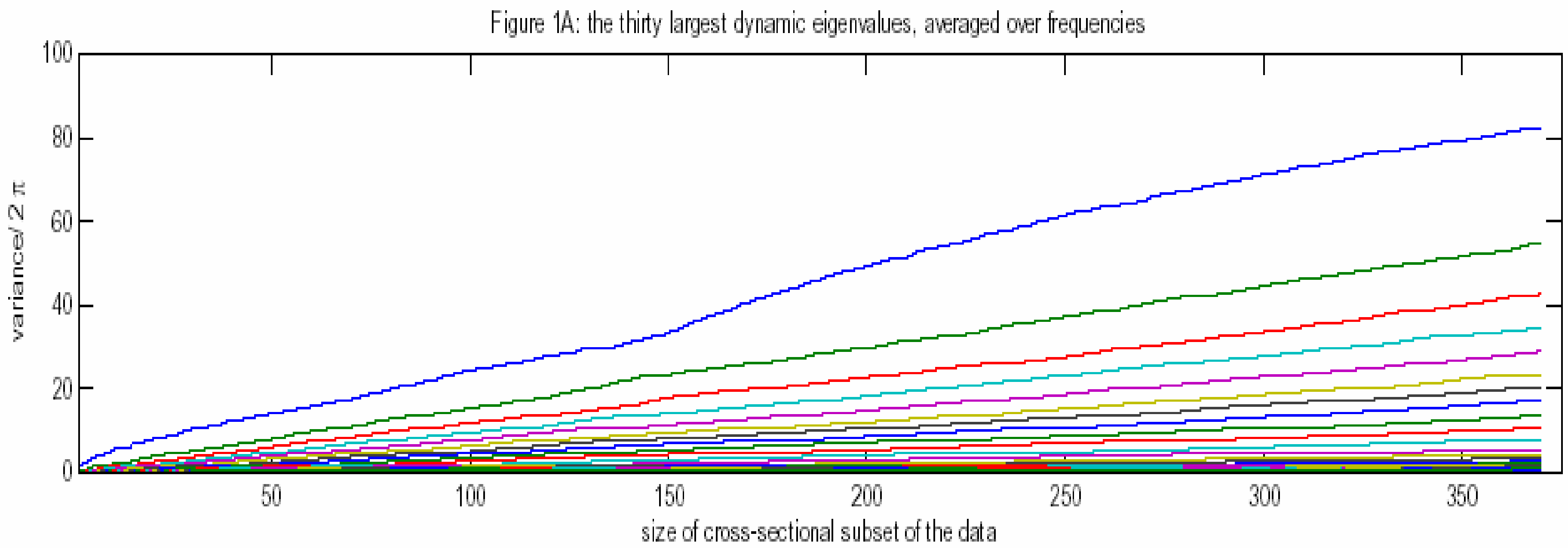
Forecasting procedure

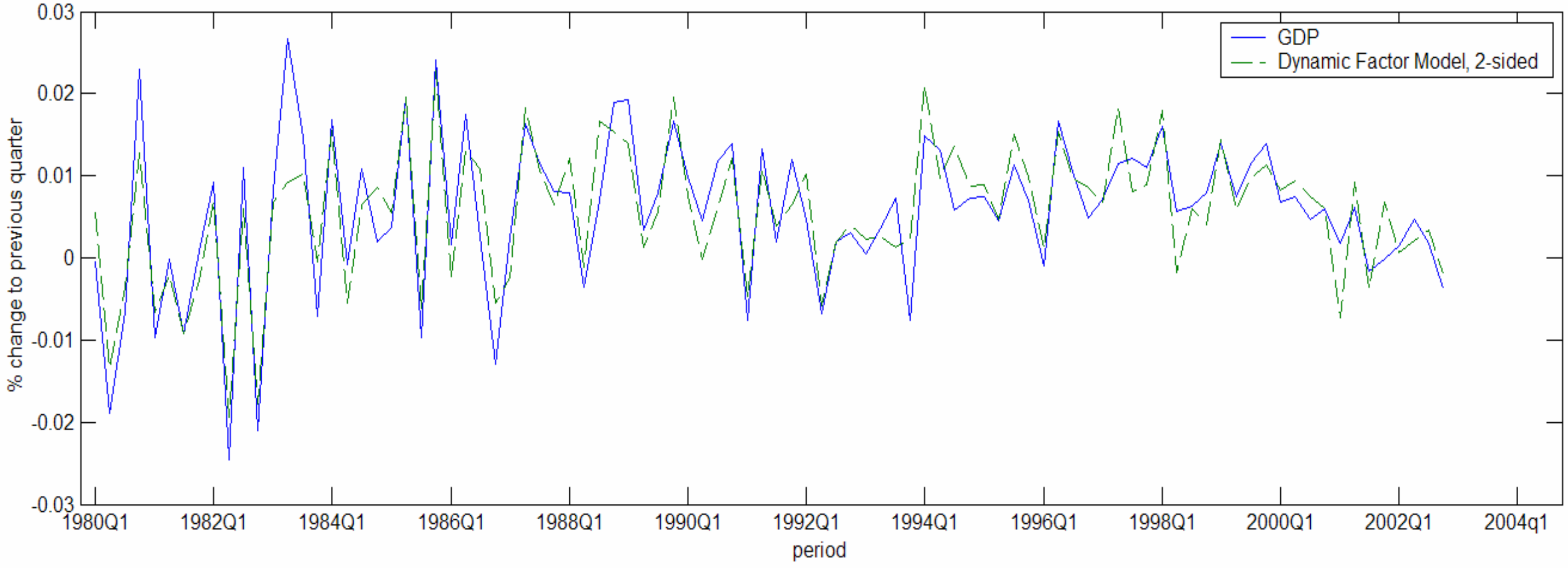
- Estimate factors
  - we use one-sided approach of Forni et al. (2002)
  - Number of factors determined by Bai/Ng Information Criteria
- Project the h-step ahead variable y_{t+h} (that is quarter-on-quarter growth rates of GDP) onto t-dated estimated factors
- Pseudo real-time forecasting exercise, starting in 1991q2 and running until 2002q4

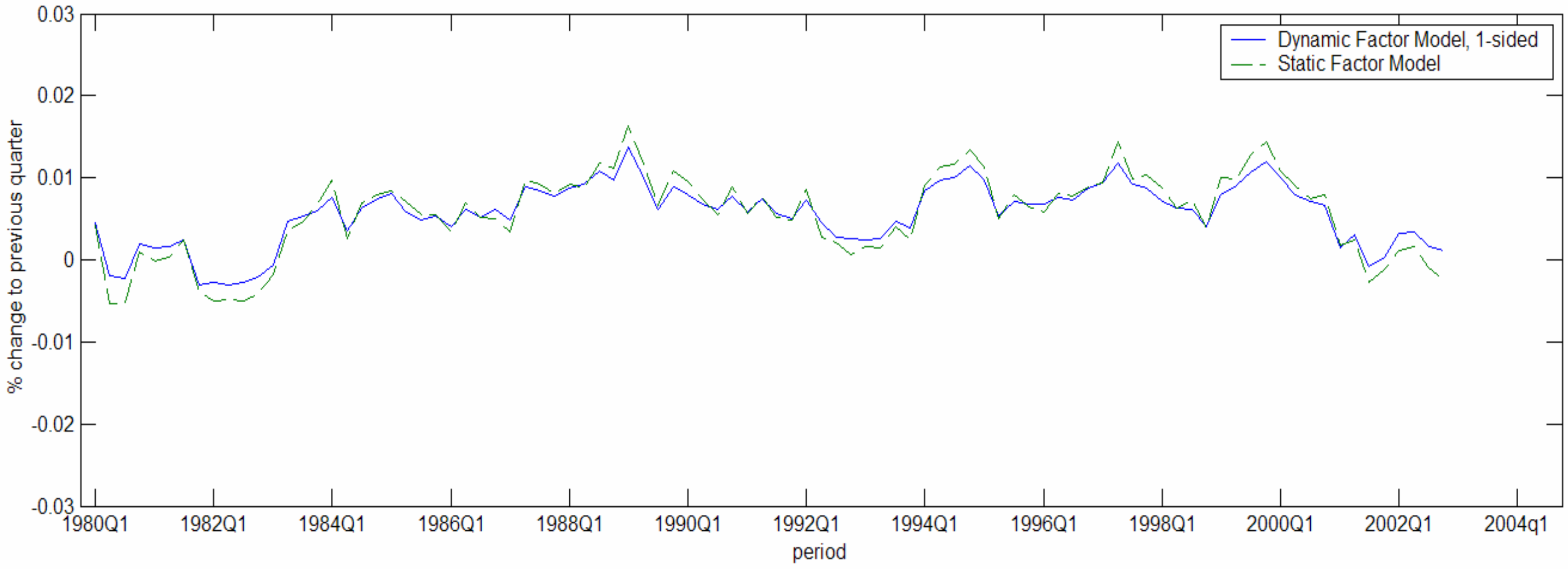
Dynamic Factor model specification

1. The average over the frequencies of the first q eigenvalues diverges, whereas the average of the (q+1)-th eigenvalue remains relatively stable

2. At r=n there should be a substantial gap between the variance explained by the q-th principal component and the variance explained by the (q+1)-th.




GDP
Dynamic Factor Model, 2-sided
% change to previous quarter
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1980Q1
1982Q1
1984Q1
1986Q1
1988Q1
1990Q1
1992Q1
1994Q1
1996Q1
1998Q1
2000Q1
2002Q1
2004q1
period


Dynamic Factor Model, 1-sided
Static Factor Model
% change to previous quarter
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1980Q1
1982Q1
1984Q1
1986Q1
1988Q1
1990Q1
1992Q1
1994Q1
1996Q1
1998Q1
2000Q1
2002Q1
2004q1
period

Empirical Results

Table 1: Relative MSE and size of the subset

forecast horizon	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
<u>full dataset</u>								
static factor model	1.05	0.86	0.93	1.1	1.02	1.11	1.32	1.34
dynamic factor model	0.99	0.92	0.98	0.99	0.96	0.97	0.97	1.03

- Boivin/Ng (2005): Size and composition of the dataset matter for factor modeling
  - Oversampling: (dominant) factors irrelevant for variable of interest
  - Data features: dispersion of the importance of the common component and serial and cross correlation in the idiosyncratic components

Composition data set matters

- We split and put ordering on the data and optimize over cross- section as well
  - Split data set into leading and lagging wrt gdp
  - Order according to cross-correlation

- Model specification according to Information Criteria in the time dimension
- Model selection according to simulated out-of-sample forecasting performance in the cross-section dimension

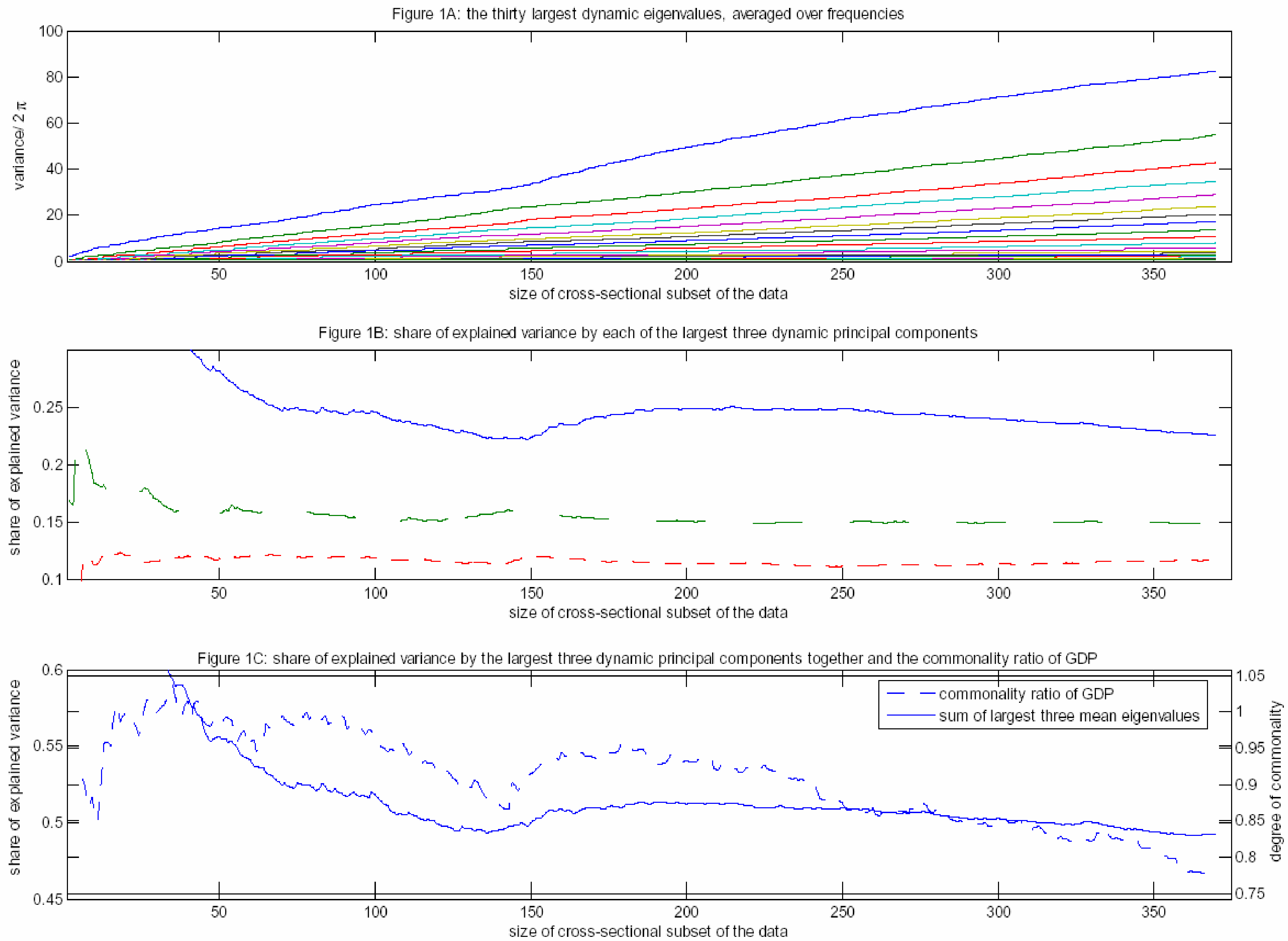
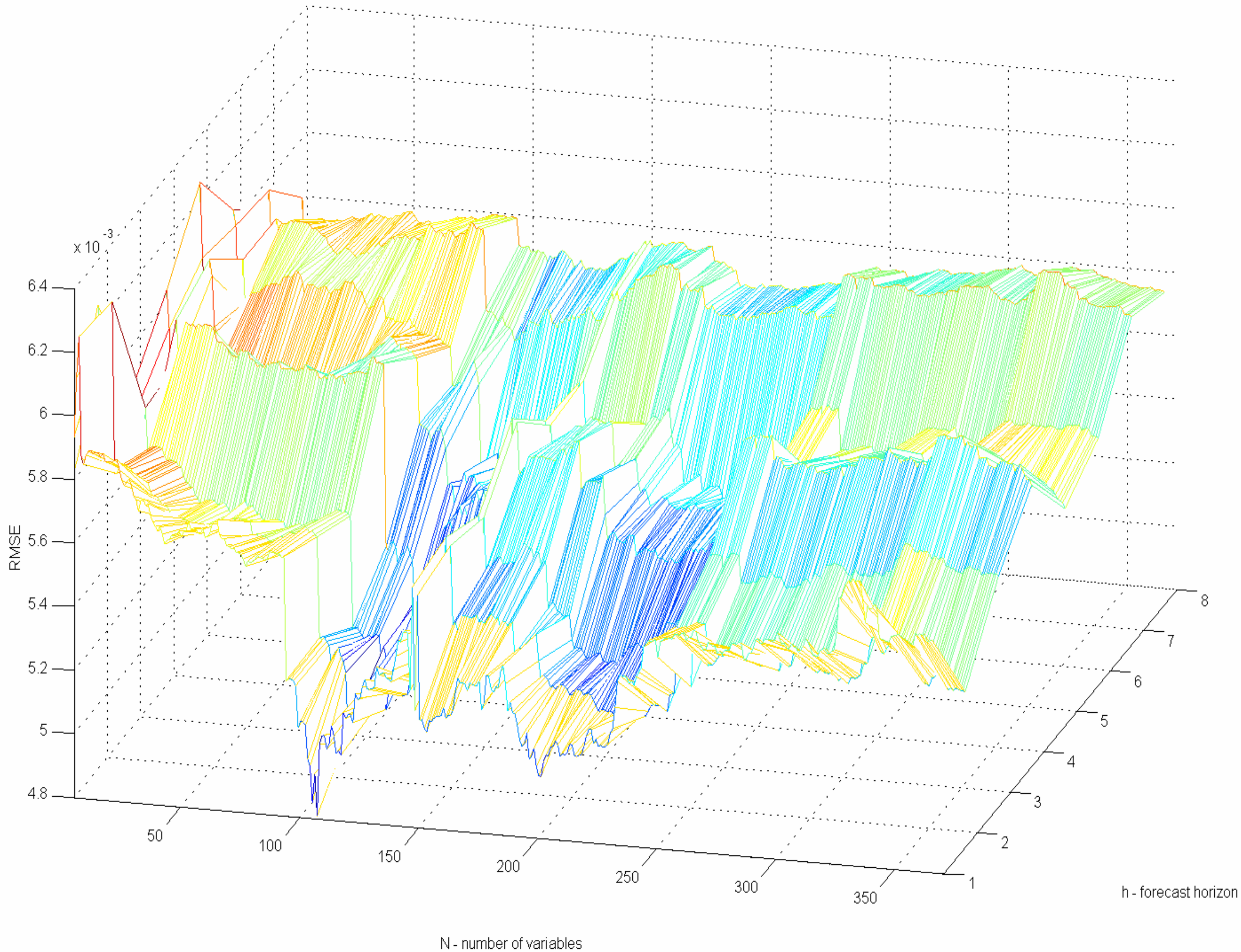


Figure 1A: the thirty largest dynamic eigenvalues, averaged over frequencies
variance/ 2 π
size of cross-sectional subset of the data
Figure 1B: share of explained variance by each of the largest three dynamic principal components
share of explained variance
size of cross-sectional subset of the data
Figure 1C: share of explained variance by the largest three dynamic principal components together and the commonality ratio of GDP
commonality ratio of GDP
sum of largest three mean eigenvalues
share of explained variance
degree of commonality
size of cross-sectional subset of the data


x 10^-3
6.4
6.2
6
5.8
5.6
5.4
5.2
5
4.8
RMSE
50
100
150
200
250
300
350
N - number of variables
1
2
3
4
5
6
7
8
h - forecast horizon

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dynamic factor model. including idiosyncratic component forecasts	1.16	0.98	1.02	0.98	0.98	0.96	1.04	1.04
subset of data								
static factor model								
size of the subset	47	370	366	177	327	367	181	190
relative mse	1.02	0.86	0.93	1.07	1.01	1.11	1.11	1.01
dynamic factor model								
size of the subset	112	113	114	115	114	114	111	199
relative mse	0.70	0.81	0.82	0.81	0.90	0.86	0.89	0.98

Conclusion

- The static approach does not seem to outperform the autoregressive benchmark (contrary to SW (2002))
- The dynamic approach outperforms the autoregressive benchmark by 10% to 30% with an optimal subset of 111-115 ordered time series
- The DFM forecasts significantly better up to 6 periods ahead according to standard forecast accuracy tests