Regional and Sectoral Dynamics of the Dutch Staffing Labour Cycle.*

Ard H.J. den Reijer†

March 15, 2011

Abstract

This study analyses the dynamic characteristics of staffing employment across different business sectors and across different geographical regions in the Netherlands. We analyse a micro data set of the market leader of the Dutch staffing employment market, i.e. Randstad. We apply the dynamic factor model to extract common information out of a large data set and to isolate business cycle frequencies with the aim of forecasting staffing and total employment. We identify regions and sectors whose cyclical developments lead the staffing labour cycle at the country level. The dynamic factor model exploits these leading characteristics at the disaggregate level to forecast the country aggregate. Finally, both dynamic and static factors turn out to be predictive summary statistics of the micro data set when employed to forecast total employment at the country level.

keywords: business cycle, dynamic factor model, disaggregate forecasting

JEL-code: C31, C53, J44, J63

1 Introduction

Flexible staffing agency work is characterized by a triangular relationship between the user firm, the employee and the private labour market intermediary

*This research projected was conducted while the author was affiliated with the central bank of the Netherlands, De Nederlandsche Bank. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Executive Boards of Sveriges Riksbank, De Nederlandsche Bank or Randstad Holding N.V. I would like to thank Randstad for providing the confidential data. Moreover, I would like to thank Lex Hoogduin, Massimiliano Marcellino, Franz Palm, Guido Schotten, anonymous referees and seminar participants at the 5th *studiedag conjunctuur* at Nijenrode University and an internal seminar for comments and suggestions. All remaining errors are mine.

†ard.den.reijer@riksbank.se, Monetary Policy Department, Sveriges Riksbank, SE-103 37 Stockholm, Sweden, tel: +46 8 787 0149.

(cf Gottfried, 1992). The staffing agency is a private matchmaker that acts as an intermediary between temporary labour supply and demand. Staffing agencies derive their income from fees charged to user firms for the temporary employment of workers registered with the agency. Staffing agencies perform the recruitment of personnel and provide a ready source of labour for their business clients. The flexible staffing industry effectively creates a spot market for labour, so user firms can replace absent employees or adjust the labour force to short-term changes and fluctuations in market demand without incurring the usual hiring and firing costs (cf. Katz and Krueger, 1999). From the perspective of the client firm, flexible staffing labour constitutes a mere variable factor of production.

Peck and Theodore (2007) and Theodore and Peck (2002) show that the American flexible staffing industry is not just a purveyor of flexibility at the micro level of meeting the needs of individual enterprises, but also at the macro level of mediating macroeconomic pressures and socioeconomic risks across the labour market as a whole. During the last 30 years, temporary employment expanded rapidly prior to macroeconomic upturns, while sharp declines in temporary employment preceded recessions. (cf. Segal and Sullivan, 1997; Theodore and Peck, 2002). Hence, fluctuations in staffing employment are timely indicators of broader business cycle motions.

Berkhout and Van Leeuwen's (2004) international comparison shows a mature Dutch flexible staffing industry that serves a relatively large part of total employment. Goldschmeding (2003), Franses and de Groot (2005*b*) and Den Reijer (2009) analyse the Dutch staffing labour market developments to monitor and forecast macroeconomic business cycles. The primary objective of this paper is to document the cyclical developments of staffing employment in the Netherlands at the disaggregate level and to identify the regions and sectors that show leading properties, (cf. Forni et al., 2001). Like Kvasnicka's (2003) German data set, the observations are directly obtained from the administrative source of a market participant instead of using survey based data. The second question is then how the disaggregate information, particularly the identified leading indicators at the sectoral and geographical level, can be exploited to forecast the country aggregate of staffing employment. The paper is structured as follows. Section 2 describes the staffing labour market and the available data set. Section 3 introduces the factor model that is employed to extract the staffing labour cycle from the data. Section 4 classifies the staffing labour cycle at the disaggregate level and identifies the leading and lagging regions and sectors. Section 5 compares different model specifications that exploit the information at the disaggregate level to forecast the staffing labour developments at the country level. Finally, section 6 employs the staffing employment data to forecast total employment.

2 Staffing agency work

The private employment agency, which is often referred to as staffing services organization[1], transforms labour from a quasi-fixed into a variable factor of production and therefore effectively create an efficient spot market for labour. The structure of the labour market and the importance of temporary and agency work differ between countries because of the legislative framework, see Berkhout and Van Leeuwen (2004) for an international comparison and Dunnewijk (2001) for a brief history in the Netherlands. The comparatively mature Dutch staffing services market grew from its inception as a percentage of the labour force from 0% in 1960 to 5% in 2004. Randstad Netherlands (Randstad hereafter) is the market leader and covers a stable market share of 40% over this entire period (cf. Franses and de Groot, 2005*a*). Randstad is the country branch of Randstad Holding[2], which is one of the largest temporary and contract staffing organizations in the world.

2.1 Data

The data set is directly obtained from the administrative source of Randstad and are nearly real-time available. The available data set consists of 1.276.393 observations on the number of contracted staffing hours. The data run from 1998 until 2005 and each year is divided into 13 subsequent administrative periods of a four week duration. Every observation consists of four dimensions; the number of staffing hours for each time period is sectorally and geographically disaggregated. The sectoral classification occurs along the four digit SBI-code and the geographical classification along the four digit system of postal codes, see appendix A for details.

At this level of disaggregation, each observation almost always corresponds to a single user firm. By nature of staffing employment, a single user firm does not make use of staffing services continuously during the entire sample period. In order to create a balanced data set, we aggregate the individual observations to the level of 15 regions and 58 sectors. The regions consist of the 12 provinces from which the agglomerations of the three largest cities are separated out. The sectors correspond to the two digit SBI-code. Now, $X_{ij,t}$ represents the

[1] The terminology of "agency work", "agency worker" and "employment agency" is practiced by the *International Confederation of Temporary Work Businesses* (CIETT). The alternative terminology of "staffing work", "staffing employee" and "staffing company" is used by the *American Staffing Association* (ASA).

[2] see http://www.randstad.com

total number of hours of staffing employment in region $i = 1, ..., 15$ and sector $j = 1, ..., 58$ during period t, running from the first period in 1998 until the second period in 2005 consisting of 92 four-weeks periods. Moreover, we create a balanced data set by deleting the combination of region i and sector j if the time series shows missing observations, that is delete X_{ij} if $\exists t$ such that $X_{ij,t} = 0$. Out of the 15*58=870 possible combinations, the balanced data set consists of N=536 different time series. Some combinations are not feasible as the type of economic activity is hardly performed in the particular region, e.g. the activity Fishing in the province Drenthe, or not present at all in the Netherlands, e.g. the activity Mining of uranium and thorium ores. The resulting balanced data set covers 97.3% of the total data set in terms of the number of observations and 98.2% in terms of the number of staffing hours. Seven sectors disappear for the balanced data set and, on the other hand, 22 sectors do not lose observations at all as a result of balancing the data set. The overall loss of roughly 2% of observations is not concentrated within a specific remaining sector, region or time period.

We calculate the aggregates of each sector i, each region j and the country total as $X_{i*,t} = \sum_j X_{ij,t}$, $X_{*j,t} = \sum_i X_{ij,t}$ and $X_t = \sum_i \sum_j X_{ij,t}$ respectively. In order to aply the dynamic factor model, all series are transformed to remove non-stationarity and corrected for outliers. The stationarity inducing transformation amounts to calculating the period-on-period growth rates[3], so we analyse $x_{ij,t} = (1 - L) \ln (X_{ij,t})$, where L is the lag operator. The time series of growth rates are corrected for outliers by replacing those observed growth rates that are more than three sample standard deviations away from the sample mean with the average of the remaining observed growth rates. In order to apply the factor method as outlined below, we construct standardized growth rates $x^s_{ij,t}$ by subtracting the sample average from the outlier corrected growth rates and dividing by the sample standard deviation[4].

3 Dynamic factor model

In order to extract the cyclical developments of staffing employment in the Netherlands at the disaggregate level and to identify the regions and sectors that show leading properties for the staffing cycle at the aggregate level, we fit a dynamic factor model to the balanced stationary data set. We apply the

[3]Considering the country total as a time series variable sampled at a quarterly frequency over the sample period 1967.1-2004.4, Franses and de Groot (2005*a*) find no evidence for a seasonal unit root performing the HEGY test statistic.

[4]Preprocessing the data by stationarity inducing transformation, outlier correction and standardisation is common practice in the literature, see Breitung and Eickmeier (2006) for an overview of factor models and their applications to economic indicators, forecasting and business cycle analysis.

methodology of Forni et al. (2000; 2001; 2001; 2004; 2005) that was developed to extract coincident and leading indicators for the euro area from a large panel of economic variables of member countries. Factor models are a tool to cope with many variables without running into problems of too little degrees of freedom often faced in regression based analysis.

Firstly, factor models summarize large data sets in few underlying forces. The extracted low-dimensional common information is then used to discern the "common signal" χ from the "idiosyncratic noise" ξ for each of the underlying variables, so

$$x^s_{ij,t} = \chi_{ij,t} + \xi_{ij,t} \tag{1}$$

The idiosyncratic motion of a variable includes the effects of local shocks that are typically sector or region specific, while the common signal affects all sectors and regions. The common component $\chi_{ij,t}$ is driven by the impact of $k = 1, ..., q$ unobserved "dynamic factors" u_{kt} that are common to all the variables in the data set.

Secondly, the dynamic factor model allows for factor loadings $\alpha_{ijk}(L)$, $k = 1, ..., q$, which describe the dynamic impact of the common dynamic factors u_{kt} on the common component:

$$\chi_{ij,t} = \alpha_{ij1}(L)\, u_{1t} + ... + \alpha_{ijq}(L)\, u_{qt}. \tag{2}$$

The common driving forces u_k can affect the individual variables with different leads and lags, which enables to classify the variables, regions and sectors as leading, coincident and lagging. The static factor model is a special case of (2) for which the factor loadings α_{ijk} only contemporaneously relate to the factors u_{kt}.

Thirdly, we further decompose the common component χ_{ijt} into a cyclical medium- and long-run component ϕ_{ijt} and a non-cyclical seasonal and irregular part ψ_{ijt}, that is

$$x^s_{ijt} = \phi_{ijt} + \psi_{ijt} + \xi_{ijt}. \tag{3}$$

This decomposition is based on a two-sided, symmetric, square summable band-pass filter $\beta(L)$, which separates waves of periodicity larger than a given critical number of periods τ :

$$\phi_{ij,t} = \sum_{k=-\infty}^{\infty} \beta_k \chi_{ij,t-k},\ \beta_k = \begin{cases} \frac{1}{k\pi}\sin(2k\pi/\tau) \text{ for } k \neq 0 \\ 1/\tau \text{ for } k = 0 \end{cases}. \tag{4}$$

The cyclical medium- and long-run component $\phi_{ij,t}$ is thereby filtered for short-run seasonal and erratic fluctuations and therefore signals more smoothly the underlying development of the staffing employment growth[5].

In order to estimate the generalized dynamic factor model, we need to specify the number of dynamic factors q, the parameter M that determines the maximum lag of auto-covariance matrix and the cyclicality parameter τ, see appendix A.4 for details. The identifying factor model assumption requires that the q largest dynamic eigenvalues diverge, whereas the remaining $N-q$ eigenvalues remain bounded as the number of time series variables N increases. We follow Forni et al.'s (2000) approach and select $q=3$ in the finite-sample, because the marginal explained variance of the q^{th} dynamic eigenvalue is larger than 10% and the $(q+1)^{th}$ one is smaller than 10%. The corresponding q dynamic eigenvectors are the estimators for the common dynamic factors u_k, $k=1,...,q$ and the dynamic factor loadings $\alpha_{ijk}(L)$ describe the dynamic impact of the k-th common factor u_k on the time series variable x_{ijt}. We use a data dependent rule to set the maximum lead and lag of M periods, that is $\alpha_{ijk,\pm n}L^{\pm n}u_{kt}=0$ for $n>M$, at $M(T)=round\left(2T^{(1/2)}\right)=19$ for our data set of $T=92$ observations in the time dimension. Finally, we set $\tau=13$, so all seasonality, which by definition entails a duration shorter than 1 year, or 13 periods, is filtered out. The medium- and long-run component then describes the cyclicality of duration longer than one year and, given the length of the sample of observations of $T=92$ periods, shorter than seven years.

Figure 1 plots the year-on-year growth rates of the total employment in the Netherlands as reported by Statistics Netherlands, the year-on-year growth rates of the country aggregate of the turnover of Randstad[6] together with the aggregate common signal $\widehat{\phi}_t$.

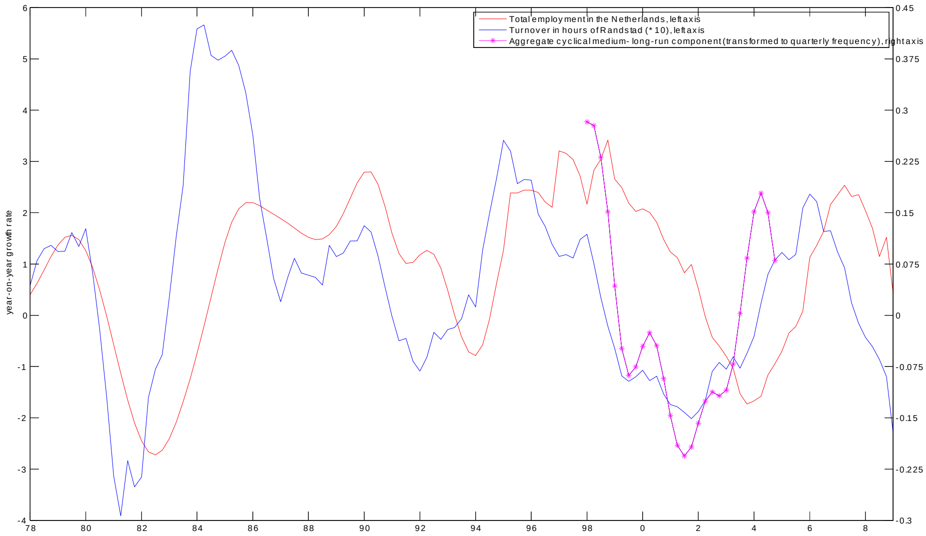
3.1 Aggregate and aggregated staffing employment

The signal at the country level ϕ_t, at the sectoral level $\phi_{i*,t}$, at the regional level $\phi_{*j,t}$ and at the disaggregate level $\phi_{ij,t}$ can be determined by projecting the corresponding aggregates x_t^s, $x_{i*,t}^s$, $x_{*j,t}^s$ and $x_{ij,t}^s$ respectively on the dynamic factors u_{kt}, which can be estimated by dynamic principal components, see appendix A.4. The linear projection of the data on the dynamic principal components provides the parameter estimates of (2), that is the factor loadings $\widehat{\alpha}_k(L), \widehat{\alpha}_{i*,k}(L), \widehat{\alpha}_{*j,k}(L)$ and $\widehat{\alpha}_{ijk}(L)$, $k=1,...,q$ respectively.

[5] CEPR´s coincident indicator of the euro area (€urocoin) reposes on a similarly composed measure that captures the cyclical signal underlying short-lived oscillations, see Altissimo et al. (2006).

[6] The history of the country aggregate of turnover data of Randstad is obtained from Franses and de Groot (2005*a*). Moreover, the staffing data from 2005 onwards orginates form the Dutch association of temporary work agencies, which covers 60 per cent of the market, see http://www.abu.nl

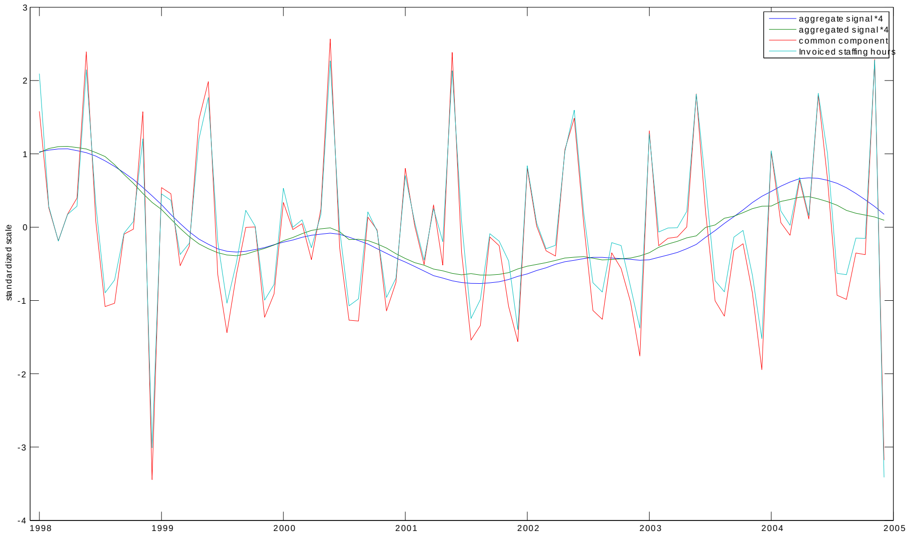
Figure 1: Growth rates of total employment, staffing employment and its cyclical component



Alternatively, the aggregated signal of the country, each sector i and each region j can be constructed as the weighted aggregate of the individual signals $\phi_{ij,t}$, that is $\overline{\phi}_{i*,t} = \sum_j a_{*j,t-1}\phi_{ijt}$, $\overline{\phi}_{*j,t} = \sum_i a_{i*,t-1}\phi_{ijt}$ and $\overline{\phi}_t = \sum_i \sum_j a_{ij,t-1}\phi_{ijt}$ respectively. The time-varying weights a_{ijt} are the shares of the individual variables in the aggregate multiplied by the ratio of the standard deviations of x_t and $x_{ij,t}$: $a_{ij,t} = \frac{\sigma_{x_{ij}}}{\sigma_x} b_{ij,t}$ with $b_{ij,t} = \frac{X_{ij,t}}{\sum_i \sum_j X_{ij,t}}$ the share of variable $X_{ij,t}$, which belongs to sector i and region j, in the total staffing turnover X_t at time t. Appendix A.2 shows that calculating the standardized growth rates x_t^s from the aggregate X_t is mathematically equivalent to aggregating the standardized growth rates of the disaggregates $x_{ij,t}^s$ using the delayed weights $a_{ij,t-1}$. However, projecting the aggregate x_t^s on the dynamic principal components is only mathematically equivalent to aggregating the projected disaggregates $x_{ij,t}^s$ if the weights are constant $\alpha_{ij,t} = \alpha_{ij}$. So, the mathematical equivalence between the aggregate signal and the aggregated signal, $\phi_t = \overline{\phi}_t$, does not hold exactly because of time varying weights.

Figure 2 shows the aggregate growth rate x_t, its aggregate common component $\widehat{\chi}_t$, the time-varying aggregated signal $\widehat{\overline{\phi}}_t$ and the aggregate signal $\widehat{\phi}_t$. The figure suggests that the aggregated signal is empirically equivalent to the aggregate signal even though time varying aggregation weights are employed.

Figure 2: Invoiced staffing hours and its model decomposition



The correlation structure of the panel of observations across both the regional and sectoral dimensions characterizes the staffing labour cycle in the Netherlands at a disaggregate level. The correlation is calculated using a sample period of seven years of available observations and is therefore only based on at most one complete business cycle. The correlation structure is summarized by the cross-correlation ρ of each individual variable's cyclical medium- and long-run component $\phi_{ij,t}$ with the aggregate cycle ϕ_t. The optimal lead l^* is determined as $l^*_{ij} = \arg\max_l |\rho_{ij}(\phi_t, \phi_{ij,t-l})|$ and its corresponding correlation $\rho^*_{ij} = \rho_{ij}\left(\phi_t, \phi_{ij,t-l^*_{ij}}\right)$. The optimal aggregate correlation and lead measures (ρ^*_{i*}, l^*_{i*}) and (ρ^*_{*j}, l^*_{*j}) are likewise obtained by employing the regional aggregate $\phi_{i*,t}$ and the sectoral aggregate $\phi_{*j,t}$ respectively.

The optimal aggregated correlation and lead measures $\left(\overline{\rho}^*_{i*}, \overline{l}^*_{i*}\right)$ and $\left(\overline{\rho}^*_{*j}, \overline{l}^*_{*j}\right)$ at the regional and sectoral level, respectively, are alternatively obtained as the weighted average of the optimal disaggregate measures, i.e. $\overline{l}^*_{i*} = \sum_j b_{*j|T}\, l^*_{ij}$ and $\overline{l}^*_{*j} = \sum_i b_{i*|T}\, l^*_{ij}$, respectively, where the weights $b_{ij|T}$ represents the variable´s share in the total staffing turnover that is constant over time: $b_{ij|T} = \frac{1}{T}\sum_{t=1}^{T} b_{ij,t}$. The empirical measures $\overline{\rho}^*$ are likewise obtained.

The following stylized example illustrates the difference between the optimal aggregate measures (ρ^*, l^*) and the aggregated optimal measures $\left(\overline{\rho}^*, \overline{l}^*\right)$.

Consider a data set of three series y_{ij}, $i = 1, 2, 3$ with weights $b_{ij} = \frac{1}{6}, \frac{1}{6}, \frac{2}{3}$ respectively. Let the correlation coefficient be $\rho_{ij}\left(y_t, y_{ij,t-l_{ij}}\right)=\frac{1}{2}$ and zero otherwise with the corresponding leads $l_{ij} = -1, 0, 1$ respectively. The weighted average correlation is $\overline{\rho}^*_{*j} = \frac{1}{2}$ and $\overline{l}^*_{*j} = -\frac{1}{6}+\frac{2}{3} = \frac{1}{2}$. The aggregated series reads as $y_{*j} = \frac{1}{6}y_{1j} + \frac{1}{6}y_{2j} + \frac{2}{3}y_{3j}$ and its optimal measures are $\rho^*_{*j} = \frac{1}{2}$ and $l^*_{*j} = 1$. The stylized example shows that the optimal aggregate measures capture the characteristics of the underlying time series variable that is most dominant in terms of weight and cross-correlation. Given equal weights and positive correlation coefficients, the underlying time series variable that shows the highest cross-correlation will be selected: $\rho^* \geq \overline{\rho}^* (\geq 0)$.

4 The empirics of staffing employment

We summarize the correlation structure across both the regional and sectoral dimensions of the staffing labour cycle as represented by the medium- and long-run signal $\widehat{\phi}$ and the year-on-year growth rate of X_{ijt}, i.e. $x^{13}_{ijt} = \left(1 - L^{13}\right)\ln\left(X_{ijt}\right)$. Christiano and Fitzgerald's (2003) band-pass filter (CF-filter) is applied to the latter growth rates to smooth away the irregularities and remaining cyclicalities with periodicity smaller than one year. The CF-filter is a finite sample approximate band-pass filter, whose parameters equal the ideal band-pass filter coefficients β_k, cf. (4), apart from the ones related to the first and last available observation. The year-on-year growth rates relate to the period-on-period growth rates as follows[7]: $x^{13}_{it} = \left(1 + L + ... + L^{12}\right)\left(\phi_{it} + \psi_{it} + \xi_{it}\right)$. So, the cross-correlation of x^{13}_{it} with another variable, say x^{13}_{kt}, involves, apart from the cross-correlation $\rho\left(\phi_{it}, \phi_{kt}\right)$, at least the auto-correlations $\rho\left(\phi_{it}, \phi_{kt\pm j}\right)$, $j = 1, ..., 12$ and the correlations between the seasonal components $\rho\left(\psi_{it}, \psi_{kt\pm j}\right)$ and the idiosyncratic components $\rho\left(\xi_{it}, \xi_{kt\pm j}\right)$, $j = 0, ..., 12$.

For $y = \left\{\widehat{\phi}, x^{13}\right\}$ the reference cycle is the aggregate series y_t and each of the series $y_{ij,t}$, $y_{i*,t}$ and $y_{*j,t}$ can be classified as pro- or counter-cyclical according to the phase angle with the reference cycle at the zero frequency[8]. Tables 1 and 2 show at the regional respectively sectoral level both the optimal aggregate results $\{l^*_{i*}, \rho^*_{i*}\}$ and the aggregated optimal results $\left\{\overline{l}^*_{i*}, \overline{\rho}^*_{i*}\right\}$ for both $\left\{\widehat{\phi}, x^{13}\right\}$. The first column of both tables reports the number of time series variables present in the corresponding region respectively sector. The subsequent two columns of both tables show respectively the weighted average of the variance

[7] Note that $\left(1 - L^{13}\right) = (1 - L)\left(1 + L + ... + L^{12}\right)$. Then, $x^{13}_{it} = \left(1 + L + ... + L^{12}\right)x_{it} = \left(1 + L + ... + L^{12}\right)\left(\phi_{it} + \psi_{it} + \xi_{it}\right)$.

[8] We recall that the cross-spectral density between two variables h and j can be expressed, in its 'polar form' as $S_{hj}(\theta) = A_{hj}(\theta)e^{-i\phi_{hj}(\theta)}$ where $\phi_{hj}(\theta)$ is the 'phase'. The phase measures the angular shift between the cosine waves of h and j at frequency θ, $-\pi < \theta \leq \pi$. At frequency zero, the phase may be either 0 or π depending on whether the long-run correlation is positive or negative, respectively.

of the common components $\overline{var}\left(\widehat{\chi}\right)$ and the variance of the aggregate common component $var\left(\widehat{\chi}\right)$. Both measures report the fraction of the variance explained by the static factor model. The difference between the two measures shows that the static factor model explains the covariation at the aggregate level much better than at the disaggregate level. The idiosyncratic motions of the variables die out in the aggregation as they are only weakly cross-correlated. The common factors explain on average 75% respectively 60% of the variation of the aggregate at the regional and sectoral level.

Table 1 reports the empirical results of the staffing labour cycle at the regional level. The four different measures for the lead almost always indicate the number of lead periods of $-6 < l^* < 6$. The regions that show a robust, but modest lead across the four different measures are Gelderland and Overijssel. The leading characteristics correspond with the relatively dominant presence of leading sectors like Wholesale and Manufacture of motor vehicles.

Table 2 reports the empirical results of the staffing labour cycle at the sectoral level. The differences across the sectors are more pronounced than across the regions according to the different statistical measures. The five sectors for which the variation is best explained by the common dynamics are: Manufacture of food products and beverages, Construction, Financial intermediation, Health and social work, and Wholesale trade and commission trade. Some sectors are more driven by idiosyncratic dynamics instead of by the common dynamics of the staffing labour cycle. Due to the hub function of the Netherlands for international freight flows in Europe, sectors like Air transport and Water transport are likely more reactive to world trade developments than to the national business cycle. The sectors Manufacture of tobacco products and Financial Intermediation show anti-cyclicality with respect to the aggregate staffing labour cycle. The four different measures of the lead period almost always indicate a number of lead periods of $-26 < l^* < 17$. The variation in leading and lagging patterns is more pronounced across sectors than across regions. The two most leading sectors in addition to some Manufacturing sectors according to the four different measures are Supporting and auxiliary transport activities, and Sale, maintenance and repair of motor vehicles and motorcycles. Retail trade shows a modest lead of less than half a year. The latter two mentioned sectors' leading properties are confirmed by business cycle analysts. The variable of issued motor vehicle permits is incorporated in Nardo et al.'s (2008) composite leading indexes for many countries. Moreover, the retail sales are part of the total sales series, which is a key indicator in The Conference Board Index, (cf. McGuckin, 2001). The five manufacturing sectors of Electrical machinery and apparatus, Machinery and equipment and Wearing apparel, Wood and products of wood and Radio, television and communication also show a modest lead of less than half a year. The three sectors that show lagging characteristics across the four different measures are Public administration, defence and compulsory social security, Insurance and pension funding and Water transport.

Table 1: The empirical results of the staffing labour cycle at the regional level.

	$\overline{var}_{i*}(\widehat{\chi})$	$var_{i*}(\widehat{\chi})$	$\overline{\rho}^*_{i*}(\widehat{\phi})$	$\overline{\rho}^*_{i*}(x^{13})$	$\rho^*_{i*}(\widehat{\phi})$	$\rho^*_{i*}(x^{13})$	$\overline{l}^*_{i*}(\widehat{\phi})$	$\overline{l}^*_{i*}(x^{13})$	$l^*_{i*}(\widehat{\phi})$	$l^*_{i*}(x^{13})$
Drenthe	0.38	0.80	0.11	0.16	0.60	0.65	-0.31	-2.62	0.00	0.00
Noord-Brabant	0.49	0.84	0.05	0.16	0.89	0.76	-3.43	-0.71	6.00	3.00
Noord-Holland	0.46	0.79	0.26	0.07	0.96	0.78	-0.75	0.85	-2.00	-4.00
Gelderland	0.54	0.96	0.51	0.38	0.77	0.85	0.04	1.11	4.00	1.00
Zuid-Holland	0.49	0.90	0.36	0.10	0.68	0.68	-2.54	2.49	0.00	-7.00
Zeeland	0.29	0.53	0.03	-0.14	-0.73	-0.45	0.12	3.03	17.00	7.00
Friesland	0.38	0.82	0.16	0.10	0.86	0.37	-5.90	-0.73	0.00	0.00
Overijssel	0.47	0.80	0.10	0.16	0.70	0.85	2.00	1.89	2.00	0.00
Flevoland	0.31	0.60	0.34	0.25	0.81	0.63	-0.35	2.42	0.00	0.00
Limburg	0.40	0.74	0.16	0.11	0.98	0.90	5.31	2.02	0.00	-2.00
Utrecht	0.51	0.77	0.11	0.31	0.95	0.74	0.66	1.59	0.00	0.00
Groningen	0.36	0.81	0.06	0.02	0.52	0.59	3.22	-1.06	0.00	-2.00
Groot-Amsterdam	0.39	0.69	0.44	0.43	0.84	0.78	2.81	1.01	1.00	-1.00
Groot-Rijnmond	0.40	0.89	0.44	0.15	0.81	0.68	0.59	1.49	5.00	-1.00
Ag.'s-Gravenhage	0.24	0.74	-0.34	-0.01	-0.86	0.53	-0.12	3.24	11.00	3.00

Notes:

The variance (var) of the common component reports the fraction of the total variance that is explained by the static factor model. The correlation ρ and the time shift l are calculated for the cycle of the corresponding region with respect to the aggregate cycle at the country level. The table reports the result for both the signal ϕ, which is extracted with the factor model, and the growth rates x^{13}. The aggregate measures without a $\overline{\text{bar}}$ summarize the results for the aggregate variables at the regional level. The aggregated measures with a $\overline{\text{bar}}$ summarize the results for the disaggregate variables. The disaggregate results are aggregated to the regional level using as aggregation weights $b_{*j|T}$.

Table 2: The empirical results of the staffing labour cycle at the sectoral level.

	$\overline{var}_{*j}\ (\widehat{\chi})$	$var_{*j}\ (\widehat{\chi})$	$\overline{\rho}^*_{*j}\ (\widehat{\phi})$	$\overline{\rho}^*_{*j}\ (x^{13})$	$\rho^*_{*j}\ (\widehat{\phi})$	$\rho^*_{*j}\ (x^{13})$	$\overline{l}^*_{*j}\ (\widehat{\phi})$	$\overline{l}^*_{*j}\ (x^{13})$	$l^*_{*j}\ (\widehat{\phi})$	$l^*_{*j}\ (x^{13})$
Agriculture, hunting and related service activities	0.29	0.70	0.19	0.10	0.87	0.88	3.22	-1.75	21.00	16.00
Manufacture of food products and beverages	0.56	0.81	-0.02	-0.05	0.71	-0.05	6.36	0.39	0.00	1.00
Manufacture of tobacco products	0.18	0.26	-0.69	-0.60	-0.74	-0.64	6.04	0.00	7.00	1.00
Manufacture of textiles	0.31	0.48	0.39	0.52	0.87	0.84	4.47	-0.54	3.00	-2.00
Manufacture of wearing apparel; dressing and dyeing of fur	0.23	0.31	0.29	0.57	0.51	0.79	4.48	4.54	8.00	7.00
Tanning and dressing of leather	0.45	0.48	-0.72	0.40	-0.72	0.40	14.00	0.00	14.00	0.00
Manufacture of wood and of products of wood and cork, except furniture	0.42	0.80	0.45	0.31	0.74	0.83	4.63	3.21	8.00	7.00
Manufacture of paper and paper products	0.32	0.69	-0.17	0.18	0.67	0.37	0.78	-0.80	5.00	3.00
Publishing, printing and reproduction of recorded media	0.35	0.73	-0.06	0.13	0.64	0.59	1.38	5.40	17.00	-1.00
Manufacture of coke, refined petroleum products and nuclear fuel	0.25	0.54	0.07	0.31	0.76	0.66	2.11	-0.80	11.00	6.00
Manufacture of chemicals and chemical products	0.44	0.84	0.28	0.09	0.68	0.54	-0.43	0.04	-1.00	-8.00
Manufacture of rubber and plastics products	0.44	0.79	0.51	0.47	0.68	0.61	4.77	1.82	8.00	0.00
Manufacture of other non-metallic mineral products	0.43	0.86	0.45	0.41	0.95	0.78	-3.13	1.75	4.00	2.00
Manufacture of basic metals	0.39	0.72	0.21	0.46	0.51	0.77	7.47	-1.06	7.00	3.00
Manufacture of fabricated metal products, except machinery and equipment	0.50	0.88	0.13	0.47	0.72	0.67	-5.31	2.77	7.00	4.00
Manufacture of machinery and equipment n.e.c.	0.42	0.84	0.29	0.49	0.45	0.73	2.67	3.36	6.00	2.00
Manufacture of office, accounting and computing machinery	0.10	0.39	0.05	0.03	-0.17	-0.04	2.94	1.03	0.00	0.00
Manufacture of electrical machinery and apparatus n.e.c.	0.34	0.66	0.50	0.36	0.67	0.66	3.65	3.10	9.00	2.00
Manufacture of radio, television and communication eq.	0.29	0.47	0.60	0.47	0.74	0.71	7.54	2.90	9.00	4.00
Manufacture of medical, precision and optical instruments	0.30	0.74	0.33	0.32	0.87	0.82	-2.94	-1.93	0.00	-6.00
Manufacture of motor vehicles, trailers and semi-trailers	0.39	0.74	0.63	0.65	0.83	0.91	0.56	-7.15	0.00	-4.00
Manufacture of other transport equipment	0.27	0.64	0.03	0.37	0.84	0.87	-1.21	3.03	8.00	2.00
Manufacture of furniture; manufacturing n.e.c.	0.29	0.62	-0.22	-0.13	0.66	-0.62	5.53	2.84	-15.00	6.00
Recycling	0.02	0.02	0.51	0.09	0.58	0.49	-2.73	-1.48	0.00	-6.00
Electricity, gas, steam and hot water supply	0.22	0.74	0.25	-0.07	-0.81	0.55	-10.61	3.70	12.00	-17.00
Collection, purification and distribution of water	0.12	0.19	0.02	0.21	-0.23	0.20	7.51	10.51	0.00	4.00
Construction	0.63	0.90	0.14	0.35	0.34	0.54	3.27	-0.52	1.00	-2.00
Sale, maintenance and repair of motor vehicles and motorcycles;	0.19	0.26	0.21	0.34	0.57	0.77	6.54	5.64	9.00	11.00
Wholesale trade and commission trade	0.59	0.90	0.35	0.20	0.85	0.68	-1.86	1.53	5.00	0.00
Retail trade, except of motor vehicles and motorcycles;	0.33	0.61	0.38	0.56	0.92	0.87	1.86	0.88	6.00	5.00
Hotels and restaurants	0.43	0.75	0.59	0.55	0.88	0.82	5.45	-1.12	2.00	3.00
Land transport; transport via pipelines	0.38	0.61	0.56	0.34	0.95	0.80	0.03	-0.81	-2.00	-12.00
Water transport	0.16	0.17	0.73	0.56	0.73	0.56	-8.00	-11.00	-8.00	-11.00
Air transport	0.18	0.15	0.78	0.48	0.79	0.49	18.90	0.11	19.00	0.00
Supporting and auxiliary transport activities	0.27	0.70	0.58	0.51	0.86	0.91	5.77	2.66	9.00	9.00
Post and telecommunications	0.25	0.33	0.55	0.29	0.87	0.55	3.63	-0.16	14.00	-2.00
Financial intermediation, except insurance and pension funding	0.61	0.88	-0.14	-0.11	-0.68	-0.26	1.73	3.11	9.00	6.00
Insurance and pension funding, except compulsory social security	0.29	0.47	0.32	0.23	0.68	0.52	-8.00	-1.44	-12.00	0.00
Activities auxiliary to financial intermediation	0.10	0.20	-0.09	0.06	-0.92	0.67	-2.29	-4.31	-25.00	2.00
Real estate activities	0.26	0.74	0.43	0.31	0.75	0.70	1.87	-1.73	8.00	1.00
Renting of machinery and equipment without operator	0.23	0.54	0.26	0.43	0.69	0.69	-0.54	0.27	4.00	0.00
Computer and related activities	0.21	0.45	0.21	0.63	0.93	0.94	-0.43	-1.35	1.00	-3.00
Research and development	0.26	0.69	0.20	0.32	0.28	0.51	3.52	1.53	3.00	0.00
Other business activities	0.39	0.67	0.00	0.16	0.78	0.56	3.76	3.26	-1.00	0.00
Public administration and defence; compulsory social security	0.55	0.80	0.31	0.24	0.93	0.69	-3.71	-2.49	-20.00	-9.00
Education	0.45	0.68	0.08	0.18	0.89	0.48	2.45	0.29	-26.00	-1.00
Health and social work	0.59	0.85	0.00	-0.52	-0.92	-0.91	-0.55	6.21	13.00	10.00
Sewage and refuse disposal, sanitation and similar activities	0.45	0.78	0.53	0.30	0.70	0.37	4.39	-0.84	6.00	-4.00
Activities of membership organizations n.e.c.	0.16	0.46	0.09	0.11	0.16	0.24	1.03	0.92	0.00	-4.00
Recreational, cultural and sporting activities	0.19	0.42	0.47	0.35	0.75	0.47	2.40	-0.68	8.00	-3.00
Other service activities	0.37	0.66	0.25	0.15	0.62	0.73	5.85	-0.64	1.00	14.00

Notes:

The variance (var) of the common component reports the fraction of the total variance that is explained by the static factor model. The correlation ρ and the time shift l are calculated for the cycle of the corresponding sector with respect to the aggregate cycle at the country level. The table reports the result for both the signal ϕ, which is extracted with the factor model, and the growth rates x^{13}. The aggregate measures without a $\overline{\text{bar}}$ summarize the results for the aggregate variables at the sectoral level. The aggregated measures with a $\overline{\text{bar}}$ summarize the results for the disaggregate variables. The disaggregate results are aggregated to the sectoral level using as aggregation weights $b_{i*|T}$.

5 Forecasting aggregate staffing employment

The natural question is how the identified leading indicators at the disaggregate level can be exploited to forecast the country aggregate of the staffing employment. This variable is closely monitored to discern patterns of the Dutch macroeconomic business cycle, cf. Goldschmeding (2003), Franses and de Groot (2005*b*) and Den Reijer (2009). As the data set is closed in the sense that the country aggregate is by definition the sum of the disaggregates, we can moreover analyse the forecasting performance from a different perspective: does forecasting the aggregate improve upon aggregating the forecasts?

We refer to the h-period ahead forecast of the growth rate of the country aggregate based on observations until time t as $x_{t+h|t}$ and to the aggregated forecast as $\overline{x}_{t+h|t} = \sum_{ij} b_{ij|t}\, x_{ij,t+h|t}$ Moreover, the different forecasts are labelled as $x^{m,fe}_{ij,t+h|t}$, where the different model specifications $m = \{SF, DF, DFC, AR, \mu\}$ employ static factors (SF), one-sided dynamic factors (DF), one-sided cyclical dynamic factors (DFC), a second order autoregressive model (AR) and the first moment of the time series (μ), respectively. The latter two model specifications are purely univariate and act as a benchmark for multivariate factor specifications.

The one-sided dynamic factors f^{DF}_t are the contemporaneous weighted cross-sectional averages of the times series variables for which the weights depend on the common-to-idiosyncratic variance ratios as determined by Forni et al.'s (2005) two-step procedure. The two-sided filters in (2) render the dynamic method less suitable for forecasting purposes, since only a poor signal can be extracted at the end of the sample. However, the two-sided filters extract the temporal variation in the data to determine the in-sample common-to-idiosyncratic variance decomposition that can be exploited in a second step to determine the one-sided dynamic factors f^{DF}_t as the contemporaneously weighted cross-sectional averages of the times series variables, see appendix A.4.1 for details. We employ Bai and Ng's (2002) information criteria (BNIC) to determine the number of $r = 2$ one-sided dynamic factors f^{DF}_t as a trade-off between the goodness-of-fit and overfitting. Finally, note that the second order autoregressive model (AR) is capable of capturing oscillatory motion. Before employing the univariate specifications $\{AR, \mu\}$, the observed time series x_{ijt} gets seasonally adjusted in an automated procedure if at least one of the seasonal dummies shows a significant coefficient at the 2.5% level.

The different forecasts $x^{m,fe}_{ij,t+h|t}$ differ not only with respect to the model specification m, but also regarding the forecast equation fe that relates the factors to the target variable. If the forecast equation $fe = \{u, fm\}$ admits the factor model structure $fe = fm$, the h-step ahead common components $\widehat{\chi}^{m}_{ij,t+h}$

are projected on the t-dated estimated factors $\widehat{f}_t^m$ and so, the autocorrelation structure of the common factor structure is exploited. The factor forecasts of the common components $\widehat{\chi}^m_{ij,T+h|T}$ are transformed to forecasts of the target variable by inverse standardization, i.e. $\widehat{x}^m_{ij,T+h|T} = \widehat{\sigma}_{ij,T}\widehat{\chi}^m_{ij,T+h|T} + \widehat{\mu}_{ij,T}$, where $\widehat{\mu}_{ij,T}$ and $\widehat{\sigma}_{ij,T}$ are the sample mean and, respectively, the standard deviation of the univariate time series variable x_{ij}. If the forecast equation $fe = \{u, fm\}$ is unrestricted $fe = u$, the h-step ahead data $x_{ij,t+h}$ are projected on the t-dated estimated factors $\widehat{f}_t^m$ and a constant. Boivin and Ng (2005) show that both approaches $fe = \{u, fm\}$ deliver identical forecasts only if the data adheres to the factor model assumptions and the model parameters are known, see also appendix A.4.3.

5.1 Results

The out-of-sample forecasting exercise starts in 2002.9 and produces 32 forecasts[9] for each horizon $h = 1, ..., 13$. We perform the forecasting exercise using both a recursive estimation window and a rolling window of a size of 61 periods[10]. The forecasting performance of the different specifications is measured by the forecast error, i.e. the difference between the forecasts and the realizations. The summary statistics are the mean squared forecast error (mse) and the variance of the forecast error (var). The difference between these two statistics is the squared mean forecast error, i.e. the bias.

Along the lines of Eickmeier and Ziegler's (2008) meta-analytic analysis of the recent macroeconomic factor forecasting literature, we report the forecasting performance along four dimensions: 1) multivariate factor models $m = \{SF, DF, DFC\}$ vs. univariate models $m = \{AR, \mu\}$; 2) free vs. factor restricted forecast equation $fe = \{u, fm\}$; 3) aggregate forecasts $x_{t+h|t}$ vs. aggregating disaggregate forecasts $\overline{x}_{t+h|t}$; 4) recursive vs. rolling window estimation. The detailed forecasting performance statistics mse and var are reported along the four dimensions for each forecast horizon h in case of the recursive window in Table A.1 vs. the rolling window in Table A.2 in Appendix A.3.

Table 3 summarizes these detailed statistics on the forecasting performance by averaging the detailed mse statistics over forecast horizon and over the relevant dimensions of the forecasting exercise. For example, the first row of table 3 reports the ratio of the mse that is the average of the multivariate specification $m = \{SF, DF, DFC\}$ against its univariate $m = \{AR, \mu\}$ equivalent. Moreover, both multivariate and univariate mse are averaged over forecast horizon h and

[9] All forecasts are incorporated in the evaluation since the country aggregate data are available until 2006.11

[10] So, the first forecasting round is based on the sample that runs from 1998.1 until 2002.9 and consists of 61 periods. The recursive and the rolling window forecasts are by construction identical in the first round.

Table 3: The relative forecasting performance of different model specifications

Forecast specification	Ratio of average mse
$\frac{\text{multivariate } m=\{SF,DF,DFC\}}{\text{univariate } m=\{AR,\mu\}}$ model	0.993
$\frac{\text{free } fe=u}{\text{factor restricted } fe=fm}$ equation	0.997
$\frac{\text{aggregate } x_{t+h\|t}}{\text{aggregated } \overline{x}_{t+h\|t}}$ forecasts	0.964
$\frac{\text{rolling}}{\text{recursive}}$ window	0.976

Notes:
The table reports the ratio of mean squared errors (mse) that are averaged over the forecast horizon $h = 1, ..., 13$ and the dimensions of the forecasting exercise. So, the mse in the first row is averaged over forecast horizon h and, moreover, over both specifications of the forecast equation fe, both aggregate and aggregated forecasts and both rolling and recursive estimation window. The results are based on the out-of-sample forecasting exercise, which starts in 2002.9 and produces 32 forecasts for each horizon.

over the three remaining dimensions of the forecasting exercise: the two specifications of the forecast equation $fe = \{u, fm\}$, aggregate $x_{t+h|t}$ and aggregated $\overline{x}_{t+h|t}$ forecasts and both rolling and recursive estimation window.

The ratio of mse of the multivariate specification versus its univariate equivalent[11] shows some value added of forecasting based on a large cross-section of data. As we will show, the slight outperformance masks substantial differences among the multivariate specifications. The ratio of mse between the two specifications of $fe = \{u, fm\}$ confirms Boivin and Ng's (2005) notion that the unrestricted forecast equation performs better. The slight outperformance again masks the observation that the outperformance is more substantial for the DF specification. The ratio of mse between the aggregate x and aggregated $\overline{x}$ forecasts shows a preference for forecasting the aggregate directly. For the recursive window method, forecasting the aggregate performs on average better than aggregating the forecasts, while the rolling window method shows no distinctive differences. One explanation for the result is that the time-varying and for outlier-corrected aggregation weights $a_{ij,t}$ might be biased, see appendix A.2. Note that forecasting the aggregate employing the factor models $m = \{SF, DF, DFC\}$ incorporates the disaggregate information as contained by the factors. So, these results confirm Hendry and Hubrich's (2006) theoret-

[11]Note that we reported the 2^{nd} order autoregressive process AR as being the best performing one. An example of an alternative specification is an $AR(p)$ process for which the lag length p is determined by the Akaike information criteria in an automated procedure. Other examples are autoregressive specifications of x_{ijt}^{13} with or without including a lagged term at $(t-13)$, i.e. $\alpha_{13}x_{ijt-13}^{13}$.

ical result on predictability that forecasting the aggregate using disaggregate information outperforms aggregating the forecasted disaggregates. Finally, the ratio of mse between the rolling vs. the recursive window shows a slight preference for the rolling window estimation. This result is consistent with Eickmeier and Ziegler's (2008) analysis of the empirical macroeconomic factor forecasting literature. Apparently, the data shows substantial temporal variation, which is accounted for by employing a rolling window scheme that allows the model specification to adapt more flexibly to the data and so overcompensates the gains from using long time series.

Table 4: The relative forecasting performance of the dynamic factor specification

	p-values				mse/mse*			
h	SF	DFC	AR	μ	SF	DFC	AR	μ
1	0.01	0.05	0.60	0.72	1.31	1.12	0.97	0.98
2	0.03	0.68	0.64	0.86	1.09	0.98	0.97	0.99
3	0.31	0.00	0.04	0.34	1.09	1.06	1.06	1.03
4	0.00	0.00	0.07	0.14	1.26	1.18	1.06	1.05
5	0.73	0.58	0.04	0.78	1.02	1.03	0.89	1.02
6	0.01	007	0.01	0.01	1.85	1.24	1.32	1.28
7	0.00	0.00	0.02	0.01	1.52	1.49	1.45	1.47
8	0.00	0.13	0.11	0.05	1.20	1.10	1.08	1.10
9	0.00	0.02	0.00	0.08	1.16	1.07	1.13	1.08
10	0.06	0.00	0.47	0.42	1.08	1.15	1.03	1.04
11	0.83	0.08	0.47	0.97	1.01	1.12	1.05	1.00
12	0.08	0.56	0.13	0.17	1.35	0.95	1.20	1.16
13	0.09	0.00	0.00	0.00	1.08	1.55	1.90	2.01

Notes:
All specifications in this table correspond to the unrestricted forecast equation $fe = u$, aggregate forecasts $x_{t+h|t}$ and rolling window estimation. The reported p-values for each forecast horizon $h = 1, ..., 13$ correspond to the Harvey, Leybourne and Newbold's (1997) small-sample modification of Diebold and Mariano's (1995) (DM) test statistic of equal forecast accuracy. Moreover, the table reports the ratio of mean squared errors (mse) of the model specification presented in the second row of the table vs. the mse^* equivalent of the dynamic factor model (DF) specification. The results are based on the out-of-sample forecasting exercise, which starts in 2002.9 and produces 32 forecasts for each horizon.

Following upon the results of table 3, the remaining analysis is based on aggregate forecasts x based on a rolling window scheme and an 6unrestricted forecast equation $fe = u$. Table 4 reports for every forecast horizon h, the relative mse and the $p-$values of the respective reported model specifications against the DF specification. The p-values correspond to Harvey, Leybourne and Newbold's (1997) small-sample modification of Diebold and Mariano's (1995) (DM) test of equal forecast accuracy. Table 4 shows that the DF specification outperforms its competitor specifications significantly so at most forecast horizons. As shown in appendix A.4.1, the DF specification nests the SF specification, which does not exploit the dynamic structure of the data. So, the outperformance by the DF model implies that the underlying data exhibit substantial dynamics. Moreover, the DFC specification nests the DF specification, which can be obtained by setting $\tau = 1$ in (4) and so, does not filter out any short-lived cyclicality. The outperformance of the DF over the DFC specification shows that exploiting only the cyclical common dynamics results in a loss of information as the observed staffing employment series exhibits a substantial seasonal pattern.

6 Application: forecasting total employment

As staffing employment is the part of the labour market that is most sensitive to the business cycle, a key question is how to relate the timely available staffing employment data to forecast total employment. Figure 1 reveals a reasonable high correlation between the Randstad staffing data and total employment, which is defined as the number of jobs and measured in full time equivalent jobs. The figure moreover suggests that cyclical turning points in total employment are preceded by their correspondents in staffing employment. In this section, we compare the performance of various forecasting models based on staffing employment with some benchmark models. Since the sample period is relatively short, we will not dedicate a part of the sample for out-of-sample forecast performance analysis, but sheerly focus on in-sample model fit. Moreover, Figure 1 shows that the sample period corresponds to diminishing and declining growth rates of total employment and therefore do not cover a complete cycle. Hence, the results in this section should be interpreted as preliminary.

Table 5 lists the regression results of various forecast specifications for the quarter-on-quarter growth rates of total employment. Specification (1) is the benchmark specification being a first order autoregressive model for quarter-on-quarter growth rates including a constant and seasonal dummies. The substantial seasonal variation in the data is captured and results in a high adjusted R^2-statistic. Specification (2) adds to the autoregressive specification (1) the indicator variable producer confidence, which shows a significantly estimated parameter. According to the release policy of Statistics Netherlands, the new releases for total employment are published 45 days after the corresponding quarter has ended, while the new releases for producer confidence are published 4 working days before the end of the corresponding month. So, we regress total employment at time t on the three month averaged quarterlized variable producer confidence, where t refers to end of the quarter in terms of data availability. The staffing data are directly obtained from the administrative source of Randstad, used to invoice firms and reimburse employees and, hence, available in nearly real-time. Specifications (3) and (4) add to the autoregressive specification (1) the quarterlized[12] Randstad country aggregate of the staffing employment and a selected part of it, respectively, which both show insignificant estimation results. The variable Randstad leading is the aggregate of a subset of the Randstad data that are procyclically leading. More precisely, we select from the underlying data set consisting of $N = 536$ series as described in section 2.1 the combinations of sector i and region j with characteristics $\{l^*_{ij}, \rho^*_{ij}\}$ relating the underlying series x^{13}_{ijt} to its aggregate x^{13}_t that imply pro-cyclicality, $\rho^*_{ij} > 0$ and positive lead time $l^*_{ij} > 0$ as described in section 4. Finally, specifications (5), (6) and (7) adds to the autoregressive leading indicator specification (2) the first two respectively static factors (SF), one-sided dynamic factors (DF) and one-sided cyclical dynamic factors (DFC) as introduced in section 5. The three factor augmented leading indicator specifications show that the two factor specifications SF and DF do contribute significantly, albeit with the second instead of the first extracted factor. In contrast to the results in section 5.1, the SF and DF factor specifications show similar performance. The underlying dynamics of the data that was exploited by the dynamic factor method apparently receded when the factors were temporally transformed from 13 periods to 4 quarters. The SF and DF factors turn out to be predictive summary statistics of the Randstad staffing data when employed to forecasting total employment, while the specifications based on only the aggregated observed staffing data do not seem to contribute to the forecast performance.

[12] The staffing employment data are sampled in 13 periods consisting of 4 weeks for every year. The transformation of 13 periods to 12 months runs according to the following algorithm: month(i)=(14-i)/13*period(i)+i/13*period(i+1) for i=1,...,12. Finally, a quarter consists of the three months summation.

Table 5: Total employment forecast specifications

Dependent variable: Total employment							
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
constant	−0.0111	−0.0105	−0.00906	−0.0111	−0.0121	−0.0122	−0.0107
	(21.66)	(15.02)	(7.08)	(13.82)	(6.03)	(6.47)	(13.03)
Total employment$_{t\text{-}1}$	0.827	0.653	0.692	0.589	0.588	0.582	0.636
	(13.78)	(7.56)	(8.51)	(4.44)	(5.78)	(5.59)	(4.96)
Producer confidence $_{*1000}$		0.343	0.278	0.32	0.431	0.428	0.33
		(2.49)	(2.22)	(2.11)	(2.93)	(2.84)	(1.82)
Randstad			0.0163				
			(1.41)				
Randstad Leading				−0.0080			
				(0.84)			
SF 1 $_{*1000}$					−0.251		
					(0.810)		
SF 2 $_{*1000}$					−1.22		
					(2.56)		
DF 1 $_{*1000}$						−0.12	
						(0.90)	
DF 2 $_{*1000}$						−0.55	
						(2.75)	
DFC 1 $_{*1000}$							0.0011
							(0.05)
DFC 2 $_{*1000}$							0.20
							(1.22)
Seasonal dummy 1	0.0139	0.0120	0.0122	0.0107	0.0149	0.0151	0.0122
	(6.24)	(4.38)	(4.44)	(3.60)	(4.68)	(4.91)	(4.74)
Seasonal dummy 2	0.0265	0.0252	0.0231	0.0262	0.0210	0.0213	0.0256
	(26.42)	(35.54)	(12.75)	(20.39)	(4.27)	(4.68)	(13.15)
Seasonal dummy 3	0.00484	0.00628	0.00254	0.00859	0.0142	0.0143	0.00661
	(5.12)	(7.91)	(0.961)	(3.16)	(4.41)	(4.72)	(5.29)
adj. R^2	0.914	0.914	0.915	0.918	0.921	0.921	0.908

Notes:

Sample period 1998q1 2004q4. The variables Total employment, Randstad and Randstad leading are specified as first differences of logs, that is in quarter-on-quarter growth rates. The variables labelled SF are static factors, DF are one-sided dynamic factors and DFC one-sided cyclical dynamic factors.

7 Conclusion

This paper analyses the developments of the staffing labour cycle in the Netherlands at a disaggregate level using the data set from Randstad. We create a balanced data set that describes the number of hours of staffing employment for 15 different regions and 58 different sectors. We apply factor models to extract low-dimensional common information from the data set and show that the extracted signal resembles the year-on-year growth rate of aggregate staffing employment. The common signal, which excludes the effects of sector or region specific shocks, is also extracted at the disaggregate level. We analyse the correlation structure and classify the disaggregate cycles as leading and lagging according to eight empirical measures. Almost all regions lead or lag the staffing labour cycle by less than half a year. The regions, whose modest leading characteristics are robust across four different empirical measures, are Gelderland and Overijssel.

Almost all sectors show a lead that lies between -2 years and +1.5 years. The differences across the sectors are more pronounced than across the regions. Three leading sectors, whose leading characteristics are robust across four different empirical measures, are Supporting and auxiliary transport activities, Sale, maintenance and repair of motor vehicles and motorcycles, and Retail trade. The turnover in the latter two sectors are known to be stylized business cycle leading indicators.

We then explored how the identified leading indicators at the disaggregate level can be exploited to forecast the country aggregate of the staffing employment. We compare different model specifications that employ static factors, dynamic factors, cyclical dynamic factors, a second order autoregressive model and the first moment of the time series. The performance is measured by the forecast bias and the mean squared forecast error. Due to substantial temporal and seasonal variation in the staffing labour market, the dynamic factor model manages to outperform the univariate benchmark forecasting models.

Finally, we employ various models based on staffing data to forecast total employment measured in number of jobs. Compared against some benchmark specifications, the final section shows that the models based on factors as summary statistics of the staffing data improve upon the forecast performance, while the models based on only the aggregated observed staffing data do not seem to contribute.

References

Altissimo, F., R. Cristadoro, M. Forni, M. Lippi and G. Veronese. 2006. New Eurocoin: Tracking Economic Growth in Real Time. Discussion Paper 5633 Center for Economic Policy Research.

Bai, J. and S. Ng. 2002. “Determining the Number of Factors in Approximate Factor Models.” *Econometrica* 70:191–221.

Berkhout, E.E. and M.J. Van Leeuwen. 2004. International Database on Employment and Adaptable Labour (IDEAL). Report 642 SEO, Research commissioned by Randstad Holding.

Boivin, J. and S. Ng. 2005. “Understanding and Comparing Factor-Based Forecasts.” *International Journal of Central Banking* 1:117–151.

Breitung, J. and S. Eickmeier. 2006. Dynamic factor models. In *Modern econometric analysis, Chapter 3*, ed. G. Huebler and J. Frohn.

Brillinger, D.R. 1981. *Time Series Data Analysis and Theory.* Springer-Verlag.

Christiano, L.J. and T.J. Fitzgerald. 2003. “The Band Pass Filter.” *International Economic Review* 44:435–465.

Den Reijer, A.H.J. 2009. “The Dutch business cycle: a finite sample approximation of selected leading indicators.” *Journal of Business Cycle Measurement and Analysis* 2009(2):89–Ű110.

Diebold, F. and R. Mariano. 1995. “Comparing Predictive Accuracy.” *Journal of Business and Economic Statistics* 13:253–263.

Dunnewijk, T. 2001. Temporary work agencies in the Netherlands: emergence and perspective. Report 1 Central Bureau of Policy Analysis.

Eickmeier, S. and C. Ziegler. 2008. “How successful are dynamic factor models at forecasting output and inflation? A meta-analytic approach.” *Journal of Forecasting* 27(3):237–265.

Forni, M., M. Hallin, M. Lippi and L. Reichlin. 2000. “The Generalized Factor Model: Identification and Estimation.” *The Review of Economics and Statistics* 82(4):540–554.

Forni, M., M. Hallin, M. Lippi and L. Reichlin. 2001. “Coincident and leading indicators for the euro area.” *The Economic Journal* 111:62–85.

Forni, M., M. Hallin, M. Lippi and L. Reichlin. 2004. “The Generalized Factor Model: Consistency and Rates.” *Journal of Econometrics* 119(2):231–255.

Forni, M., M. Hallin, M. Lippi and L. Reichlin. 2005. "The Generalized Dynamic Factor Model: One-Sided Estimation and Forecasting." *Journal of the American Statistical Association* 100(471):830–840.

Forni, M. and M. Lippi. 2001. "The Generalized Dynamic Factor Model: Representation Theory." *Economic Theory* 17:1113–1141.

Franses, P.H.B.F. and E.A. de Groot. 2005*a*. Real time estimates of GDP growth. Econometric Institute Report 1 Erasmus University Rotterdam.

Franses, P.H.B.F. and E.A. de Groot. 2005*b*. Real time estimates of GDP growth, based on two-regime models. Econometric Institute Report 32 Erasmus University Rotterdam.

Goldschmeding, F.J.D. 2003. Ervaringen in de uitzendbranche. Een casus: Randstad. In *Motoriek van de economie. Hoe kan het bedrijfsleven inspelen op economische ontwikkelingen?*, ed. E.A. de Groot.

Gottfried, H. 1992. "In the margins: flexibility as a mode of regulation in the temporary help service industry." *Work, Employment and Society* 6:443–460.

Harvey, D., S. Leybourne and P. Newbold. 1997. "Testing the Equality of Prediction Mean Squared Errors." *International Journal of Forecasting* 13:281–291.

Hendry, D.F. and K.S.E.M. Hubrich. 2006. Forecasting Economic Aggregates by Disaggregates. Working Paper 589 European Central Bank.

Katz, L.F. and A.B. Krueger. 1999. "The High-Pressure U.S. Labor Market of the 1990s." *Brookings Papers on Economic Activity* 1:1–65.

Kvasnicka, M. 2003. Inside the black box of temporary help agencies. Discussion Paper Humboldt University Berlin.

McGuckin, R.H. 2001. *Business Cycle Indicators Handbook*. The Conference Board.

Nardo, M., M. Saisana, A. Saltelli, S. Tarantola, A. Hoffman and E. Giovannini. 2008. *Handbook on constructing composite indicators: methodology and user guide*. Organisation for Economic Co-operation and Development. ISBN 978-92-64-04345-9.

Peck, J. and N. Theodore. 2007. "Flexible recession: the temporary staffing industry and mediated work in the United States." *Cambridge Journal of Economics* 31(2):171–192.

Segal, L.M. and D.G. Sullivan. 1997. "The Growth of Temporary Services Work." *Journal of Economic Perspectives* 11:117–136.

Stock, J. and M.W. Watson. 2002. "Forecasting Using Principal Components From a Large Number of Time Predictors." *Journal of the American Statistical Association* 97:1167–1179.

Theodore, N. and J. Peck. 2002. "The temporary staffing industry: growth imperatives and limits to contingency." *Economic Geography* 78(4):463–494.

A Appendix

A.1 Data

The data set consists of 1.276.393 observations on the number of contracted staffing hours employed by Randstad. An observation consists of the number of paid hours of contracted staffing agency work. The number of paid hours is larger than the number of invoiced hours since for instance holiday hours and sickness leave are included in addition to worked hours. Every observation consists of four dimensions: the number of paid hours of contracted staffing agency work, the time dimension denoted by the year and the period, the sector to which the user company belongs and the geographical area where the user firm is located. So, the sector and location refers to the company at which the staffing agency worker is employed. The user firm that hires a staffing employee is located in a geographical area that is classified according to the 4-digit postal code. The data consists of 2882 different postal codes that can be aggregated to 466 different municipalities and 15 different regions. The time dimension of the data is divided into years and periods. Every year consists of 13 subsequent administrative periods of 4 weeks. Observations are available from the first period of 1998 until and including the first period of 2005.

Statistics Netherlands (*Centraal Bureau voor de Statistiek,* CBS) employs a systematic hierarchical classification system for economic activities called the *Standaard Bedrijfsindeling* (SBI). On the highest 2-digits level of compartments, the classification consists of 58 different sectors[13], which are compatible with the International Standard Industrial Classification of All Economic Activities (ISIC).

Statistics Netherlands employs a systematic hierarchical classification system for regional units called the *Nomenclatuur van Territoriale eenheden voor de Statistiek* (NUTS). The system provides a non-overlapping countrywide division of the Netherlands in 40 regional units (NUTS3), which can be aggregated to 12 provinces (NUTS2). In this study, we employ 15 regions that correspond to the 12 provinces from which the agglomerations of the three largest cities are separated out. The three NUTS3 regional units *Groot-Amsterdam, Groot-Rijmond* and *Agglomeratie ´s Gravenhage* correspond with the agglomerations of the three biggest cities Amsterdam, Rotterdam and the Hague, respectively.

[13]Compartments describing activities that are however not present in the Netherlands are for instance the "mining of uranium and thorium ores" and the "mining of metal ores".

A.1.1 Employment data

Statistics Netherlands publishes quarterly sampled data on total employment both in number of hours and in number of full time equivalent (fte) jobs. Due to a benchmark revision in 2005, only historical data back until the first quarter of 2001 are available. However, regarding the number of fte jobs, a real-time data set is published on the website of the euro area business cycle network[14]. The final series is constructed backwards by employing year-on-year growth rates.

A.2 On aggregation

The balanced data set consists of the series $X_{ij,t}$, which are the total number of hours of staffing employment in region $i = 1, ..., 15$ and sector $j = 1, ..., 58$ during period t, running from 1998.1 until 2005.1 consisting of 92 four-weeks periods. The time series of aggregate staffing employment at the regional level consists of $X_{i*,t} = \sum_j X_{ij,t}$, the sectoral level equivalent $X_{*j,t} = \sum_i X_{ij,t}$ and the time series of total staffing employment in the Netherlands equals $X_t = \sum_i \sum_j X_{ij,t}$. Let the time-varying shares of the individual variables in the aggregate be defined as: $\alpha_{ij,t} = \frac{X_{ij,t}}{\sum_i \sum_j X_{ij,t}}$. Lemma (1) shows that calculating the growth rates x_t of the aggregate X_t is equivalent to aggregating the growth rates of the disaggregates $x_{ij,t}$ using the delayed weights $\alpha_{ij,t-1}$.

Lemma 1 *Let* $X_t = X_{1,t} + X_{2,t}$ *be a time series variable. Let* x_t *be the growth rate of* X_t *and let* $b_{i,t} = \frac{X_{i,t}}{X_t}$. *Then* $x_t = b_{1,t-1}x_{1,t} + b_{2,t-1}x_{2,t}$

Proof. $\Delta \ln (X_t) \approx x_t = \frac{\Delta X_t}{X_{t-1}} = \frac{\Delta X_{1,t} + \Delta X_{2,t}}{X_{1,t-1} + X_{2,t-1}} = \frac{\Delta X_{1,t}}{X_{1,t-1}} \frac{X_{1,t-1}}{X_{1,t-1} + X_{2,t-1}}$
$+ \frac{\Delta X_{2,t}}{X_{2,t-1}} \frac{X_{2,t-1}}{X_{1,t-1} + X_{2,t-1}} = x_{1,t} \left(\frac{X_{1,t-1}}{X_{1,t-1} + X_{2,t-1}} \right) + x_{2,t} \left(\frac{X_{2,t-1}}{X_{1,t-1} + X_{2,t-1}} \right) = b_{1,t-1}x_{1,t} +$
$b_{2,t-1}x_{2,t}$. ■

The dynamic factor model decomposition is only defined for data sets that consist of a panel of stationary time series.

Lemma 2 *Let* $X_t = X_{1,t} + X_{2,t}$ *be a first-order integrated,* $I(1)$, *time series variable. For* $i = 1, 2$ *let* x_t *and* $x_{i,t}$ *be the corresponding growth rates,* x_t^s *and* $x_{i,t}^s$ *the standardized growth rates with means* μ_x *and* μ_{x_i} *and standard deviations* σ_x *and* σ_{x_i} *respectively. Let the time-varying weights be* $b_{i,t} = \frac{X_{i,t}}{X_t}$. *Then* $x_t^s = a_{1,t-1}x_{1,t}^s + a_{2,t-1}x_{2,t}^s$ *with* $a_{i,t} = b_{i,t}\frac{\sigma_{x_i}}{\sigma_x}$.
Proof. *Lemma (1) shows that* $\sigma_x x_t^s + \mu_x = b_{1,t-1}\left(\sigma_{x_1}x_{1,t}^s + \mu_{x_1}\right)$
$+b_{2,t-1}\left(\sigma_{x_2}x_{2,t}^s + \mu_{x_2}\right)$. *Rewriting this equation leads to* $x_t^s = b_{1,t-1}\frac{\sigma_{x_1}}{\sigma_x}x_{1,t}^s +$ $b_{2,t-1}\frac{\sigma_{x_2}}{\sigma_x}x_{2,t}^s$ *and* $\mu_x = b_{1,t-1}\mu_{x_1} + b_{2,t-1}\mu_{x_2}$. ■

[14] http://www.eabcn.org/eabcn-real-time-database

Projecting the aggregate x_t^s on the estimated factors is only mathematically equivalent to aggregating the projected disaggregates $x_{ij,t}^s$ if the weights are constant $a_{ij,t} = a_{ij}$. Let x_{it}^s denote a vector of T observations for $i = 1, 2$ and u a $(T \times q)$ −matrix of q common components u_{it}. Say, you have $x_t^s = a_1 x_{1t}^s + a_2 x_{2t}^s$, then $\chi_t^{'} = \left(u_t^{'} u_t\right)^{-1} u_t^{'} x_t^s u_t^{'} = \left(u_t^{'} u_t\right)^{-1} u_t^{'} \left(a_1 x_{1t}^s + a_2 x_{2t}^s\right) u_t^{'} = a_1 \left(u_t^{'} u_t\right)^{-1} u_t^{'} x_1^s u_t^{'} + a_2 \left(u_t^{'} u_t\right)^{-1} u_t^{'} x_{2t}^s u_t^{'} = a_1 \chi_{1t}^{'} + a_2 \chi_{2t}^{'}$.

The assumptions needed for this mathematical equivalence are not necessarily met in practice. Firstly, the weights a_i are time varying. Secondly, lemma (1) does not hold precisely since the disaggregate growth rates $x_{i,t}$ and the aggregate growth rate x_t are independently corrected for outliers. Thirdly, lemma (2) only holds in population as the estimated variances may vary over sample, i.e. $\widehat{\sigma}_{x|1,...,t} \neq \widehat{\sigma}_{x|1,...,t-1}$.

A.3 Empirical results

The detailed results of the forecasting exercise are reported along four dimensions: multivariate factor models $m = \{SF, DF, DFC\}$ vs. univariate models $m = \{AR, \mu\}$, free vs. factor restricted forecast equation $fe = \{u, fm\}$, aggregate forecasts $x_{t+h|t}$ vs. aggregated disaggregate forecasts $\overline{x}_{t+h|t}$ and recursive vs. rolling window estimation. The forecasting performance along the first three dimensions are reported in Table A.1 and Table A.2 employing recursive respectively rolling window estimation. The first column reports for each forecast horizon $h = 1, ..., 13$ the forecasting performance of the AR-model. The upper part of the column reports the *mse* and the lower part the *var* of the forecast errors. The other columns report the forecasting performance of the respective models as a ratio to the performance of the benchmark AR-model.

A.4 The Estimator

In this appendix, we show in more detail how the common component χ can be estimated in a stepwise procedure. Moreover, we will highlight the parameter condition that makes the static factor model a special case of the dynamic one. Finally, we show in more detail the estimator for the factor model forecasts in case the forecast equation is restricted to admit the factor model structure, i.e. $fe = fm$.

A.4.1 The dynamic method

The dynamic method as outlined in Forni et al. (2000; 2001; 2001; 2004; 2005) (FHLR) consists of the frequency-domain counterpart of the static method. The dynamic factors $\mathbf{u}_t = (u_{1t}...u_{qt})'$ are estimated by the dynamic principal components, which are the static principal components of the spectral density matrix as outlined by Brillinger (1981). Denote by $\mathbf{X}^{nT} = (x_{it})_{i=1...n,t=1...T}$ an $n \times T$ rectangular of observations[15], which are realisations of real-valued stationary stochastic processes $\left\{\mathbf{x}_t = (x_{1t}...x_{nt})'\right\}$. Let $\widehat{\mathbf{\Gamma}}_{\mathbf{X}}^{nT}(k) = \frac{1}{T-k}\sum_{t=k+1}^{T}\mathbf{x}_t^{nT}\mathbf{x}_{t-k}^{nT'}$ be the k-lag sample covariance of $\mathbf{x}_t^{nT}$. FHLR suggest the following stepwise procedure:

(i) estimate the spectral density matrix (cf. Brillinger, 1981) of $\mathbf{X}^{nT}$ as $\sum_{\mathbf{X}}^{nT}(\theta_h) = \sum_{k=-M}^{M} \widehat{\Gamma}_{\mathbf{X}}^{nT}(k)\,\omega_k e^{-ik\theta_h}$, $\theta_h = 2\pi h/(2M+1)$, $h = 0,...,2M$, where $\omega_k = 1-|k|/(M+1)$ is the Bartlett window of size M. Like Forni et al. (2000), we set $M(T)=$
$round\left(2T^{(1/2)}\right)$ such that the convergence rate is $M(T)/T = O\left(T^{(1/2)}\right)$;

(ii) calculate from the spectral density matrix $\sum_{\mathbf{X}}^{nT}(\theta_h)$ the q largest dynamic eigenvalues $\lambda_j^{nT}(\theta_h)$ and the corresponding dynamic eigenvectors $\mathbf{p}_j^{nT}(\theta_h)$, $j = 1,...,q$.for $h = 0,...,2M$. We follow Forni et al.'s (2000) approach and select $q = 3$ in a finite-sample such that the marginal explained variance of the q^{th} dynamic eigenvalue is larger than 10% and the $(q+1)^{th}$ equivalent is smaller than 10%;

(iii) let $\underline{\mathbf{p}}_q^{nT}(\theta_h) = \left(\mathbf{p}_1^{nT'}(\theta_h)...\mathbf{p}_q^{nT'}(\theta_h)\right)'$ the $(q \times n)-$matrix of dynamic eigenvectors and $\underline{\lambda}_q^{nT}(\theta_h)$ a diagonal matrix with the q largest dynamic eigenvalues on the diagonal. Inverse Fourier transformation of $\widehat{\sum}_{\chi}^{nT}(\theta_h) = \underline{\mathbf{p}}_q^{nT'*}(\theta_h)$ $\underline{\lambda}_q^{nT}(\theta_h)\ \underline{\mathbf{p}}_q^{nT}(\theta_h)$ ($*$ denotes complex conjugate) results in the correlation matrix of the common component $\widehat{\Gamma}_{\chi}^{nT}(k) = \frac{1}{(2M+1)}\sum_{k=-M}^{M}\widehat{\sum}_{\chi}^{nT}(\theta_h)\ \omega_k e^{ik\theta_h}$ for $h = 0,...,2M$. Moreover, the estimated common dynamic factors are $\widehat{\boldsymbol{u}}_t^{nT} = \frac{1}{(2M+1)}\sum_{k=-M}^{M}\sum_{h=0}^{2M}\underline{\mathbf{p}}_q^{nT}(\theta_h)\ e^{ik\theta_h}\mathbf{x}_{t-k}^{nT}$. Projecting the data on the common dynamic factors gives the estimator of the cyclical medium- and long-run common component:

$$\widehat{\phi}_{nt}^{nT} = \frac{1}{(2M+1)}\sum_{k=-M}^{M}\sum_{h=0}^{2M}\beta_k\underline{\mathbf{p}}_q^{nT'*}(\theta_h)\,\underline{\mathbf{p}}_q^{nT}(\theta_h)\,e^{ik\theta_h}\mathbf{x}_{t-k}^{nT}, \qquad \text{(A.1)}$$

where the band-pass filter coefficients $\beta(L)$ are defined in (4). The finite sample approximation consists of truncating the tails of the filter that involve unavailable data observations, so $\beta_k = 0$ for $k > M$.

[15] The notation is simplified in that the time series variables are indexed by the single subscript i instead of by the sector-region coordinates ij as in the main text. Moreover, the additional superscript nT denotes the sample size.

(iv) repeat step (iii) using the $(q+1)$ to n ordered eigenvalues to obtain $\widehat{\Gamma}_{\xi}^{nT}(k)$;

(v) let $\widehat{\mathbf{S}}^{nT}=\left(\widehat{S}_1^{nT\prime}...\widehat{S}_r^{nT\prime}\right)^{\prime}$ the $(r\times n)$-matrix containing the r generalized eigenvectors of the couple of matrices $\left(\widehat{\Gamma}_{\chi}^{nT}(0),\widehat{\Gamma}_{\xi}^{nT}(0)\right)$ with the normalization that $\widehat{S}_i^{nT\prime}diag\left(\widehat{\Gamma}_{\xi}^{nT}(0)\right)\widehat{S}_j^{nT\prime}=1$ if $i=j$ and zero otherwise. We use Bai and Ng's (2002) information criteria (BNIC) to determine the r generalized static factors as a trade-off between the goodness-of-fit and overfitting. The factors can then be estimated by the generalized principal components, i.e. $\widehat{\mathbf{F}}^{nT}=\widehat{\mathbf{S}}^{nT}\mathbf{X}^{nT}$, with $\widehat{\mathbf{F}}^{nT}=\left(\widehat{\mathbf{f}}_1^{nT}...\widehat{\mathbf{f}}_T^{nT}\right)$ a $(r\times T)-$matrix of the stacked estimated factors;

(vi) let $\widetilde{\chi}_{i,T+h|T}^{nT}$ be the h-step ahead factor forecasts of the common component of the i-th variable given T observations of n time series variables. The forecasts for the dynamic common component can be obtained by projecting the $(t+h)-$dated unobserved common component $\widehat{\chi}_{t+h}^{nT}$ on the t-dated factors $\widehat{\mathbf{f}}_t^{nT}$, which for variable i results in:

$$\widetilde{\chi}_{i,T+h|T}^{nT}=\left[\widehat{\Gamma}_{\chi}^{nT}(h)\right]_i\widehat{\mathbf{S}}^{nT\prime}\left(\widehat{\mathbf{S}}^{nT}\widehat{\Gamma}_{\mathbf{X}}^{nT}(0)\widehat{\mathbf{S}}^{nT\prime}\right)^{-1}\widehat{\mathbf{S}}^{nT}\mathbf{X}^{nT} \quad \text{(A.2)}$$

Evidently, the in-sample estimator for the common component can be obtained by setting $h=0$.

Step (i) until step (iii) allow to estimate the dynamic factor model. The estimated cyclical common component $\widehat{\phi}_{nt}$ is calculated by applying time filters to the x´s before averaging along the cross-section. The dynamic estimation method consists of two-sided filters and cannot be applied at the end of the sample, which is the most important part for forecasting. By truncating the time filters, the performance of the estimator deteriorates as t approaches T. Therefore in step (v), FHLR construct generalized principal components $\mathbf{F}^{nT}$, which are contemporaneous averages of $\mathbf{X}^{nT}$ that minimize the ratio of the variance of the idiosyncratic to common component.

A.4.2 The static method

Evidently, Stock and Watson's (2002) static factors obtain as a special case of $\widehat{\mathbf{F}}^{nT}$ with $M=0$ and assuming in step (v) that $diag\left(\widehat{\Gamma}_{\xi}^{nT}(0)\right)=I_N$, i.e. the identity matrix. Computing the generalized principal components of $\mathbf{x}_{nt}$ is equivalent to computing the standard principal components of $\mathbf{y}_{nt}=H\mathbf{x}_{nt}$ with $\det(H)\neq 0$ and H such that $H\xi_{nt}\xi_{nt}^{\prime}H^{\prime}$ is the $n\times n$-identity matrix. When the idiosyncratic variance-covariance matrix is diagonal, the normalization amounts to dividing each of the x´s by the standard deviation of its idiosyncratic component.

A.4.3 Factor based forecasting

If the forecast equation $fe = \{u, fm\}$ admits the factor model structure $fe = fm$, the h-step ahead forecasts for the common component $\widetilde{\chi}^{nT}_{i,T+h|T}$ is obtained by (A.2). The forecasts for the target variable are then obtained by inverse standardization, i.e. $\widehat{y}_{i,T+h|T} = \widehat{\sigma}_{iT}\widehat{\chi}^{m}_{ij,T+h|T} + \widehat{\mu}_{iT}$, where $\widehat{\mu}_{iT}$ and $\widehat{\sigma}_{iT}$ are the sample mean and, respectively, the standard deviation of the ith variable y_i[16].

If the forecast equation $fe = \{u, fm\}$ is unrestricted $fe = u$, the h-step ahead forecasts are obtained from an unrestricted forecast equation: $\widehat{y}_{i,T+h|T} = \widehat{\mu}_{iT,h} + \widehat{\theta}_{iT,h}\widehat{\mathbf{f}}_T$. The parameters $\widehat{\mu}_{iT,h}$, $\widehat{\theta}_{iT,h}$ are obtained by a linear regression of $x_{i,t+h}$ on the estimated factors $\widehat{\mathbf{f}}_t$ and a constant. The orthogonal projection of the $(t+h)-$dated variable $y_{i,t+h}$ on the t-dated factors results in $\theta_{iT,h} = \sigma_{iT}\left[\Gamma^{nT}_{\mathbf{X}}(h)\right]\mathbf{S}^{nT\prime}\left(\mathbf{S}^{nT\prime}\Gamma^{nT}_{\mathbf{X}}(0)\,\mathbf{S}^{nT\prime}\right)^{-1}$. This expression for $\theta_{iT,h}$ differs in two respects from its equivalent for the restricted factor model (A.2). First, the data are directly projected on the estimated factors, i.e. $\Gamma^{nT}_{\mathbf{X}}(h)$ instead of imposing the factor decomposition (1) and therefore only project the common component $\widehat{\Gamma}^{nT}_{\chi}(h)$. Second, projecting $y_{i,t+h}$ instead of its standardized equivalent $x_{i,t+h}$ makes the corresponding standard deviation σ_{iT} appear in the expression.

So as shown by Boivin and Ng (2005), the two different forecast equations $fe = \{u, fm\}$ would provide identical forecasts in population if the data admits the factor decomposition (1). Moreover, identical forecasts in sample would require that the estimated constant of the linear regression equals the in-sample standard deviation: $\widehat{\mu}_{iT,h} = \widehat{\mu}_{iT}$ and that the estimate for the parameter of the linear regression model $\theta_{iT,h}$ equals: $\widehat{\theta}_{iT,h} = \widehat{\sigma}_{iT}\left[\widehat{\Gamma}^{nT}_{\mathbf{X}}(h)\right]_i \widehat{\mathbf{S}}^{nT\prime}\left(\widehat{\mathbf{S}}^{nT}\widehat{\Gamma}^{nT}_{\mathbf{X}}(0)\,\widehat{\mathbf{S}}^{nT\prime}\right)^{-1}$.

[16] So, we refine the notation and introduce the time series variable y_i whose standardized equivalent is x_i.

Table A.1: Forecasting performance of different models using recursive windows

h	$x^{AR,u}_{t+h\|t}$	$x^{\mu,u}_{t+h\|t}$	$x^{SF,u}_{t+h\|t}$	$x^{SF,fm}_{t+h\|t}$	$x^{DF,u}_{t+h\|t}$	$x^{DF,fm}_{t+h\|t}$	$x^{DFC,u}_{t+h\|t}$	$x^{DFC,fm}_{t+h\|t}$	$\overline{x}^{AR,u}_{t+h\|t}$	$\overline{x}^{\mu,u}_{t+h\|t}$	$\overline{x}^{SF,u}_{t+h\|t}$	$\overline{x}^{SF,fm}_{t+h\|t}$	$\overline{x}^{DF,u}_{t+h\|t}$	$\overline{x}^{DF,fm}_{t+h\|t}$	$\overline{x}^{DFC,u}_{t+h\|t}$	$\overline{x}^{DFC,fm}_{t+h\|t}$
	mse	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio
1	0.75	1.04	1.39	1.36	1.09	1.08	1.13	1.07	1.04	1.04	1.36	1.37	1.05	1.04	1.09	1.06
2	0.75	1.03	1.09	1.08	0.92	0.92	1.05	1.05	1.01	1.03	1.08	1.08	0.91	0.91	1.04	1.04
3	0.75	1.02	1.07	1.06	0.98	0.98	1.04	1.04	1.01	1.02	1.06	1.06	0.98	0.98	1.04	1.04
4	0.75	1.01	1.20	1.20	0.97	0.97	1.07	1.02	0.98	1.01	1.19	1.20	0.95	0.95	1.08	1.02
5	0.80	0.98	1.00	0.98	0.89	0.89	0.97	0.98	0.97	0.99	0.99	0.99	0.90	0.90	0.98	0.99
6	0.80	0.97	1.35	1.30	0.77	0.79	0.95	0.99	0.88	0.98	1.32	1.32	0.77	0.79	0.96	0.99
7	0.75	1.00	0.98	1.03	0.63	0.68	0.80	1.02	0.93	1.00	1.07	1.04	0.64	0.69	0.82	1.03
8	0.75	0.99	1.06	1.05	0.90	0.92	1.01	1.00	1.01	1.00	1.06	1.07	0.90	0.92	1.02	1.01
9	0.77	0.99	1.01	1.02	0.86	0.89	0.99	0.99	0.98	0.99	1.04	1.04	0.85	0.89	0.99	1.00
10	0.74	1.03	1.05	1.06	0.98	0.98	1.05	1.04	1.05	1.04	1.06	1.07	0.98	0.99	1.06	1.05
11	0.78	0.97	1.00	1.00	0.97	0.96	1.10	0.98	0.96	0.98	1.01	1.01	0.98	0.97	1.09	0.99
12	0.74	0.96	1.07	1.10	0.82	0.83	0.90	0.97	0.65	0.97	1.09	1.09	0.79	0.84	0.86	0.98
13	0.79	1.00	0.55	0.56	0.52	0.74	0.60	0.99	0.61	1.00	0.54	0.54	0.52	0.75	0.60	0.99
h	var	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio
1	0.75	1.04	1.39	1.36	1.09	1.08	1.13	1.06	1.04	1.04	1.36	1.37	1.05	1.04	1.09	1.06
2	0.75	1.03	1.09	1.08	0.92	0.92	1.05	1.04	1.01	1.03	1.08	1.08	0.91	0.90	1.04	1.04
3	0.75	1.02	1.07	1.06	0.98	0.98	1.04	1.04	1.01	1.02	1.06	1.06	0.98	0.97	1.04	1.04
4	0.75	1.01	1.21	1.20	0.97	0.97	1.07	1.02	0.98	1.01	1.19	1.20	0.95	0.95	1.08	1.02
5	0.79	0.99	1.00	0.98	0.89	0.89	0.98	0.99	0.96	0.99	0.99	0.99	0.91	0.90	0.98	0.99
6	0.78	0.97	1.37	1.32	0.79	0.80	0.97	0.99	0.85	0.97	1.33	1.32	0.78	0.79	0.97	0.99
7	0.73	1.01	1.00	1.04	0.63	0.69	0.82	1.03	0.91	1.01	1.08	1.05	0.64	0.69	0.83	1.03
8	0.73	1.01	1.08	1.06	0.92	0.93	1.01	1.02	0.99	1.01	1.06	1.07	0.92	0.93	1.02	1.02
9	0.76	1.00	1.02	1.03	0.87	0.90	1.00	1.00	0.97	1.00	1.04	1.04	0.86	0.90	1.00	1.00
10	0.73	1.04	1.06	1.07	0.99	0.99	1.07	1.04	1.04	1.04	1.07	1.07	0.99	0.99	1.07	1.04
11	0.76	0.99	1.01	1.01	0.98	0.97	1.11	0.99	0.95	0.99	1.02	1.02	0.98	0.97	1.10	0.99
12	0.73	0.97	1.08	1.11	0.81	0.84	0.88	0.98	0.64	0.97	1.10	1.10	0.78	0.83	0.84	0.98
13	0.79	1.00	0.54	0.55	0.51	0.74	0.56	0.99	0.60	1.00	0.54	0.54	0.51	0.75	0.56	0.99

Notes:
$x^{m,fe}_{t+h|t}$, represents the h-step ahead forecast with the different model specifications $m = \{SF, DF, DFC, AR, \mu\}$. The specifications employ respectively static factors, dynamic factors, cyclical dynamic factors, 2^{nd} order autoregressive model and the first moment of the time series. Moreover, the forecast equation fe $= \{u, fm\}$ is unrestricted such that the time series variables are forecasted directly, or, respectively, admits the factor model structure such that the common component is forecast. The first column reports for each forecast horizon h the forecast performance of the AR-model. The upper part of the column reports the mean squared error (mse) and the lower part the variance (var) of the forecast errors. The other columns report the forecasting performance of the respective models as a ratio to the performance of the benchmark AR-model. The forecasting exercise starts in 2002.9 and produces 32 forecasts for each horizon. All forecasts are evaluated since the aggregate data at the country level are available until 2006.4. The forecasts are generated using a recursive estimation window.

Table A.2: Forecasting performance of different models using rolling windows

h	$x^{AR,u}_{t+h\|t}$	$x^{\mu,u}_{t+h\|t}$	$x^{SF,u}_{t+h\|t}$	$x^{SF,fm}_{t+h\|t}$	$x^{DF,u}_{t+h\|t}$	$x^{DF,fm}_{t+h\|t}$	$x^{DFC,u}_{t+h\|t}$	$x^{DFC,fm}_{t+h\|t}$	$\overline{x}^{AR,u}_{t+h\|t}$	$\overline{x}^{\mu,u}_{t+h\|t}$	$\overline{x}^{SF,u}_{t+h\|t}$	$\overline{x}^{SF,fm}_{t+h\|t}$	$\overline{x}^{DF,u}_{t+h\|t}$	$\overline{x}^{DF,fm}_{t+h\|t}$	$\overline{x}^{DFC,u}_{t+h\|t}$	$\overline{x}^{DFC,fm}_{t+h\|t}$
	mse	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio
1	0.76	1.01	1.35	1.35	1.03	1.03	1.16	1.07	1.01	1.02	1.32	1.34	1.00	1.01	1.13	1.06
2	0.75	1.02	1.12	1.11	1.03	1.02	1.01	1.04	1.02	1.02	1.10	1.09	1.01	1.00	1.00	1.04
3	0.79	0.97	1.03	1.02	0.94	0.94	0.99	0.98	0.96	0.96	1.02	1.02	0.93	0.93	0.99	0.98
4	0.77	0.99	1.19	1.19	0.94	0.95	1.11	1.02	0.98	1.00	1.16	1.17	0.94	0.94	1.11	1.02
5	0.70	1.14	1.14	1.14	1.12	1.10	1.16	1.16	1.11	1.16	1.15	1.15	1.13	1.11	1.16	1.16
6	0.81	0.98	1.41	1.36	0.76	0.77	0.94	0.98	0.94	0.97	1.35	1.33	0.77	0.78	0.96	0.98
7	0.73	1.02	1.05	1.05	0.69	0.73	1.03	1.04	1.00	1.02	1.09	1.06	0.70	0.74	1.00	1.04
8	0.74	1.02	1.11	1.10	0.93	0.95	1.02	1.04	1.03	1.02	1.10	1.11	0.93	0.96	1.01	1.04
9	0.80	0.96	1.04	1.03	0.89	0.90	0.95	0.97	0.97	0.97	1.03	1.03	0.88	0.90	0.94	0.97
10	0.76	1.01	1.05	1.05	0.97	0.97	1.11	1.01	1.01	1.02	1.04	1.05	0.96	0.97	1.10	1.02
11	0.82	0.95	0.96	0.96	0.95	0.92	1.07	0.94	0.97	0.93	0.96	0.96	0.95	0.92	1.02	0.94
12	0.72	0.97	1.12	1.12	0.83	0.87	0.79	1.00	0.82	0.98	1.10	1.10	0.83	0.87	0.78	1.00
13	0.76	1.06	0.57	0.57	0.53	0.76	0.81	1.05	0.90	1.06	0.56	0.55	0.52	0.76	0.79	1.05
h	var	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio	ratio
1	0.76	1.02	1.36	1.35	1.04	1.03	1.16	1.07	1.01	1.03	1.33	1.34	1.01	1.01	1.13	1.07
2	0.75	1.02	1.12	1.11	1.02	1.01	1.01	1.04	1.02	1.02	1.10	1.09	1.00	1.00	1.00	1.04
3	0.79	0.97	1.03	1.02	0.94	0.93	0.99	0.98	0.96	0.96	1.02	1.02	0.92	0.93	0.99	0.98
4	0.77	0.99	1.18	1.18	0.94	0.95	1.11	1.02	0.98	1.00	1.16	1.16	0.93	0.94	1.10	1.01
5	0.70	1.13	1.12	1.11	1.11	1.09	1.13	1.14	1.10	1.14	1.12	1.12	1.12	1.10	1.12	1.14
6	0.80	0.97	1.38	1.34	0.74	0.75	0.92	0.96	0.93	0.96	1.32	1.30	0.75	0.76	0.93	0.95
7	0.70	1.04	1.08	1.07	0.69	0.74	1.05	1.05	1.02	1.04	1.11	1.08	0.71	0.75	1.02	1.05
8	0.72	1.02	1.10	1.10	0.92	0.95	1.01	1.03	1.02	1.01	1.09	1.10	0.93	0.95	1.00	1.02
9	0.79	0.95	1.01	1.01	0.88	0.89	0.92	0.94	0.96	0.95	1.01	1.01	0.86	0.88	0.91	0.94
10	0.75	1.01	1.04	1.04	0.96	0.96	1.11	1.00	1.01	1.02	1.03	1.04	0.95	0.96	1.09	1.00
11	0.80	0.96	0.97	0.97	0.94	0.91	1.07	0.93	0.97	0.93	0.96	0.96	0.93	0.91	1.03	0.93
12	0.71	0.96	1.11	1.11	0.79	0.84	0.75	0.99	0.79	0.98	1.08	1.08	0.79	0.85	0.75	0.98
13	0.76	1.06	0.56	0.57	0.52	0.75	0.79	1.04	0.89	1.05	0.54	0.54	0.50	0.76	0.76	1.04

Notes:

The forecasts are generated using a rolling estimation window of 61 periods. See the footnote of the previous table for further explanation.